Midterm 2 Review B **Name:** ____________________
Date: 23/10/2022
Math 54: Fall 2022

Problem 1

True/False. Justify your answers.

1. If A is row equivalent to B they have the same eigenvalues.
2. If A is non-invertible, it has at least one zero eigenvalue.
3. Let $T : V \to V$ be a linear transformation and $\mathcal{B}, \mathcal{C}$ be two bases for V. Then $A_{B,B}$ and $A_{C,C}$ have the same eigenvalues.

Problem 2

Let

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

Compute A^5.

Problem 3

True/False. Justify your answers.

1. If λ is an eigenvalues of A, it has geometric multiplicity at least 1.
2. If an $n \times n$ matrix A has an eigenvalue λ with geometric multiplicity n, then A is a diagonal matrix.
3. Any eigenvector of a matrix A is in the column space of A.
4. If a square matrix A is diagonalisable and invertible, then A^{-1} is also diagonalisable.

Problem 4

Suppose V and W are vector spaces and $T : V \to W$ is an invertible linear map. Suppose $\mathcal{B} = \{\mathbf{v}_1, ..., \mathbf{v}_n\}$ is a basis of V . Show that $\mathcal{S} = \{T(\mathbf{v}_1), ..., T(\mathbf{v}_n)\}$is a basis of W.

Problem 5

Find a basis for the $\text{Col}(A)^{\perp}, \text{Col}(B)^{\perp}$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 0 \end{bmatrix}, \ B = \begin{bmatrix} 1 & 3 \\ 0 & 1 \\ 2 & 0 \end{bmatrix}$$

Problem 6

Let W be a subspace of $\mathbb{R}^n$. Show that

$$W^{\perp} = \{\mathbf{u} \in \mathbb{R}^n \text{ such that } \mathbf{u} \cdot \mathbf{w} = 0 \text{ for all } \mathbf{w} \in W\}$$

is a subspace of $\mathbb{R}^n$.

Problem 7

True/False. Justify your answers.

1. $\mathbf{u} \cdot \mathbf{v} - \mathbf{v} \cdot \mathbf{u} = 0$
2. If $\|\mathbf{u}\|^2 + \|\mathbf{v}\|^2 = \|\mathbf{u} + \mathbf{v}\|^2$, then $\mathbf{u}$ and $\mathbf{v}$ are orthogonal.
3. For an $m \times n$ matrix A, vectors in the null space of A are orthogonal to vectors in the row space of A.

Problem 8

Determine if the sets of vectors is orthonormal. If it is only orthogonal, normalise the vectors to produce an orthonormal set.

$$\begin{bmatrix} \sqrt{2} \\ 3 \\ 3 \end{bmatrix}, \begin{bmatrix} 6 \\ -\sqrt{2} \\ -\sqrt{2} \end{bmatrix}, \begin{bmatrix} 0 \\ -\sqrt{10} \\ \sqrt{10} \end{bmatrix}$$

Problem 9

Determine whether each of these sets are an orthogonal basis for $\mathbb{R}^3$

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} \right\}, \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 4 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} \right\}$$

Problem 10

Find the distance of the point $\mathbf{x} = \begin{bmatrix} 3 \\ 2 \\ 5 \end{bmatrix}$ from the two dimensional subspace $W \subset \mathbb{R}^3$ spanned by

$\mathbf{u}_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$, $\mathbf{u}_2 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

Problem 11

True/False. Justify your answers.

1. Not every orthogonal set in $\mathbb{R}^n$ is linearly independent.
2. If the columns of an $m \times n$ matrix A are orthonormal, then $\|\mathbf{x}\| = \|A\mathbf{x}\|$.
3. The orthogonal projection of $\mathbf{y}$ onto $\mathbf{v}$ is the same as the orthogonal projection of $\mathbf{y}$ onto $c\mathbf{v}$ whenever $c \neq 0$.
4. A matrix with orthonormal columns is invertible.

Problem 12

Find an orthogonal basis for the null space of

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{bmatrix}$$