Midterm 2 Review A **Name:** ____________________
Date: 23/10/2022
Math 54: Fall 2022

Problem 1

What conditions must a set $A \subset B$ satisfy to be a subspace of B?

Problem 2

Consider a linear operator $T : V \to W$ is a subspace. Is the null space of T a subset of V or of W? Show that it is a subspace of V or W.

Problem 3

For each of the following, show that the set of vectors is a basis and find the coordinate of the following vectors in their respective bases:

1. $\mathbf{v} = \begin{bmatrix} 1 \\ 4 \\ -3 \end{bmatrix}, \ \mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}$
2. $\mathbf{v} = 1 + x + x^2, \ \mathcal{B} = \left\{ 1, 1 - 2x, x - 2x^2 \right\}$

Problem 4

Let $T : \mathbb{R}^5 \to \mathbb{R}^3$ and rank of T be 2. What is the nullity of T?

Problem 5

Find the change of basis matrix $P_{C \leftarrow B}$

$$\mathcal{B} = \left\{ \begin{bmatrix} 0 \\ -2 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \\ 3 \end{bmatrix}, \begin{bmatrix} 7 \\ 1 \\ 1 \end{bmatrix} \right\}, \mathcal{C} = \left\{ \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 5 \\ 4 \\ 0 \end{bmatrix} \right\}$$

Problem 6

Find the matrix $A_{B,C}$ of the linear transformation T relative to the bases $\mathcal{B}, \mathcal{C}$. Is this transformation one-to-one or onto?

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right) = \begin{bmatrix} x_1 + 2x_2 \\ 5x_1 - x_2 \\ 6x_1 + 3x_2 \end{bmatrix}, \mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}, \mathcal{C} = \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} \right\}$$

Problem 7

Consider the map $T : \mathbb{P}^3 \to \mathbb{P}^2$ where $T(f) = f'$.

1. Find the matrix of T with respect to the standard bases.
2. Find bases for $\mathbb{P}^3, \mathbb{P}^2$ such that the matrix for T relative to these bases is

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

3. Is this map one-to-one or onto?

Problem 8

Find the determinants of the following matrices

$$A = \begin{bmatrix} 2 & -4 \\ 5 & -1 \end{bmatrix}, \; B = \begin{bmatrix} 1 & -4 & -2 \\ 0 & 5 & -1 \\ 0 & 0 & 7 \end{bmatrix}, \; C = \begin{bmatrix} 1 & 1 & -2 \\ 1 & 2 & -1 \\ 1 & 0 & -1 \end{bmatrix}$$

Problem 9

Let $T : V \to V$ be a linear transformation. What does it mean for $\mathbf{v}$ to be an eigenvector of T?

Problem 10

Consider $T : \mathbb{P}^2 \to \mathbb{P}^2$ defined by $T(f) = f(0) + f' \cdot x + f(0) \cdot x^2$

1. Is $f(x) = 1$ an eigenvector of this transformation? If so what is the corresponding eigenvalue?
2. Is $f(x) = x^2$ an eigenvector of this transformation? If so what is the corresponding eigenvalue?

Problem 11

What is the characteristic polynomial for a square matrix A?

Problem 12

Find the characteristic polynomial and all eigenvalues of the following matrix. Is the matrix diagonalisable? If so diagonalise it.

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Problem 13

Compute the following given that

$$\mathbf{u} = \begin{bmatrix} 3 \\ -4 \\ -5 \end{bmatrix}, \ \mathbf{v} = \begin{bmatrix} 6 \\ -2 \\ 3 \end{bmatrix}$$

1. $\mathbf{u} \cdot \mathbf{v}$
2. $\text{proj}_{\mathbf{u}}\mathbf{u}$
3. $\text{proj}_{\mathbf{u}}\mathbf{v}$
4. $\text{proj}_{\mathbf{v}}\mathbf{v}$

Problem 14

Find the distance between the two vectors

$$\mathbf{u} = \begin{bmatrix} 3 \\ -4 \\ -5 \end{bmatrix}, \ \mathbf{v} = \begin{bmatrix} 6 \\ -2 \\ 3 \end{bmatrix}$$

Problem 15

Use the Gram-Schmidt process to find a set of orthonormal vectors that have the same span as $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix} \right\}$.