Quiz # 9 **Name:** Solutions
Date: 10/11/2022
Math 54: Fall 2022

Problem 1 - 5 Points

Consider the matrix

$$A = \begin{bmatrix} 4 & -2 \\ 2 & -1 \\ 0 & 0 \end{bmatrix}$$

1. Find all singular values of the matrix.
2. Find the singular value decomposition of the matrix.

$$\underline{\underline{A}}^T\underline{\underline{A}} = \begin{pmatrix} 4 & 2 & 0 \\ -2 & -1 & 0 \end{pmatrix} \begin{pmatrix} 4 & -2 \\ 2 & -1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 20 & -10 \\ -10 & 5 \end{pmatrix}$$

Char. poly. : $(20-\lambda)(5-\lambda) - 100 = 0$

$\lambda^2 - 25\lambda = 0, \quad \lambda = 0, \lambda = 25$

→ Singular values are $0, \sqrt{25} = 0, 5$.

For eigenvectors of $\underline{\underline{A}}^T\underline{\underline{A}}$: $\lambda = 0, \quad \underline{v} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$\lambda = 25, \quad \underline{v} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$

So $\underline{\underline{\Sigma}} = \begin{pmatrix} 0 & \\ & 5 \end{pmatrix}, \quad \underline{\underline{V}} = \frac{1}{\sqrt{5}} \begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}$

Problem 2 - 5 Points

Consider the following ODE:

$$y''(t) + 4y'(t) + 4y = 0$$

1. Find the general solution to the above equation.
2. Find the solution of the equation for $y(0) = 1, y'(0) = 0$.

1) AE: $m^2 + 4m + 4 = 0$, $(m+2)^2 = 0$, $m = -2, -2$

GS: $y(t) = Ae^{-2t} + Bte^{-2t}$

2) $y'(t) = -2Ae^{-2t} + Be^{-2t} - 2Bte^{-2t}$

$y(0) = A = 1$

$y'(0) = -2A + B = 0 \longrightarrow B = 2$

$\Rightarrow y(t) = e^{-2t} + 2te^{-2t}$