Quiz # 6 **Name:** Solutions
Date: 13/10/2022
Math 54: Fall 2022

Problem 1 - 5 Points

Determine if the following matrices are diagonalisable: 1) $\begin{bmatrix} 2 & -1 \\ 0 & -3 \end{bmatrix}$ 2) $\begin{bmatrix} 2 & 0 & 0 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$

1) Eigenvalues are 2, -3 distinct

— Yes. Diagonalisable

---

2) $\det(\underline{\underline{A}} - \lambda \underline{\underline{I}}) = \det \begin{pmatrix} 2-\lambda & 0 & 0 \\ 1 & 1-\lambda & -1 \\ 1 & -1 & 1-\lambda \end{pmatrix} = (2-\lambda) \begin{vmatrix} 1-\lambda & -1 \\ -1 & 1-\lambda \end{vmatrix}$

$= (2-\lambda)\left[(1-\lambda)^2 - 1\right] = (2-\lambda)\left[\lambda^2 - 2\lambda\right] = 0$

$\Rightarrow (2-\lambda)^2 \lambda = 0$, $\lambda = 0$, multiplicity 1
$\lambda = 2$, multiplicity 2

for $\lambda = 2$: $\underline{\underline{A}} - 2\underline{\underline{I}} = \begin{pmatrix} 0 & 0 & 0 \\ 1 & -1 & -1 \\ 1 & -1 & -1 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

has 2 free variables, so dim nullspace $(\underline{\underline{A}} - 2\underline{\underline{I}}) = 2$

equal to its algebraic multiplicity

$\longrightarrow$ Yes. Diagonalisable.

Problem 3 - 5 Points

Diagonalise the following matrix

$$A = \begin{bmatrix} 3 & -2 \\ 1 & 1 \end{bmatrix}$$

$$\det(\underline{\underline{A}} - \lambda \underline{\underline{I}}) = \det\begin{pmatrix} 3-\lambda & -2 \\ 1 & 1-\lambda \end{pmatrix} = (3-\lambda)(1-\lambda) + 2 = 0$$

$$\Rightarrow \lambda^2 - 4\lambda + 5 = 0$$

$$(\lambda-2)^2 + 1 = 0 \quad , \quad \lambda = 2 \pm i$$

For $\lambda = 2+i$, $\underline{\underline{A}} - \lambda \underline{\underline{I}} = \begin{pmatrix} 3-(2+i) & -2 \\ 1 & 1-(2+i) \end{pmatrix} = \begin{pmatrix} 1-i & -2 \\ 1 & -1-i \end{pmatrix}$

$$\underset{R_2 \times (1-i)}{\sim} \begin{pmatrix} 1-i & -2 \\ 1-i & -2 \end{pmatrix} \sim \begin{pmatrix} 1-i & -2 \\ 0 & 0 \end{pmatrix} , \Rightarrow \begin{array}{l} (1-i)x_1 - 2x_2 = 0 \\ x_2 = x_2 \end{array}$$

$$\Rightarrow \begin{array}{l} x_1 = \frac{2}{1-i} x_2 \\ x_2 = x_2 \end{array} , \quad \frac{2}{1-i} = \frac{2}{1-i} \cdot \frac{1+i}{1+i} = \frac{2+2i}{2} = 1+i \quad , \Rightarrow \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = x_2 \begin{pmatrix} 1+i \\ 1 \end{pmatrix}$$

For $\lambda = 2-i$, $\underline{\underline{A}} - \lambda \underline{\underline{I}} = \begin{pmatrix} 3-(2-i) & -2 \\ 1 & 1-(2-i) \end{pmatrix} = \begin{pmatrix} 1+i & -2 \\ 1 & -1+i \end{pmatrix}$

$$\underset{R_2 (1+i)}{\sim} \begin{pmatrix} 1+i & -2 \\ 1+i & -2 \end{pmatrix} \sim \begin{pmatrix} 1+i & -2 \\ 0 & 0 \end{pmatrix} \rightarrow \begin{array}{l} (1+i)x_1 - 2x_2 = 0 \\ x_2 = x_2 \end{array}$$

$$\begin{array}{l} x_1 = \frac{2}{1+i} x_2 \\ x_2 = x_2 \end{array} , \quad \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = x_2 \begin{pmatrix} 1-i \\ 1 \end{pmatrix} \Rightarrow$$

$$\underline{\underline{A}} = \underline{\underline{P}}\,\underline{\underline{D}}\,\underline{\underline{P}}^{-1}$$

$$\underline{\underline{P}} = \begin{pmatrix} 1+i & 1-i \\ 1 & 1 \end{pmatrix} , \quad \underline{\underline{D}} = \begin{pmatrix} 2+i & 0 \\ 0 & 2-i \end{pmatrix}$$