Quiz # 5 **Name:** Solutions
Date: 6/10/2022
Math 54: Fall 2022

Problem 1 - 6 Points

Fix bases

$$B = \{1, x, x^2\}, C = \{1 + x^3, 2x + 3x^2, 2 + 5x^2 + x^3, x^2\}$$

for $\mathbb{P}_2(\mathbb{R})$ and $\mathbb{P}_3(\mathbb{R})$, respectively. Let T be the linear transformation

$$T : \mathbb{P}_2(\mathbb{R}) \to \mathbb{P}_3(\mathbb{R})$$

with associated matrix

$$A_{B,C} = \begin{bmatrix} 3 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & -1 \\ 2 & 0 & 0 \end{bmatrix}$$

Calculate the polynomial $T(3 - 2x + 2x^2)$. You must give your answer as a polynomial.

$$[f]_B = [3 - 2x + 2x^2]_B = \begin{pmatrix} 3 \\ -2 \\ 2 \end{pmatrix}$$

$$A_{B,C}[f]_B = \begin{pmatrix} 3 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & -1 \\ 2 & 0 & 0 \end{pmatrix} \begin{pmatrix} 3 \\ -2 \\ 2 \end{pmatrix} = \begin{pmatrix} 11 \\ 1 \\ -4 \\ 6 \end{pmatrix} = [T(f)]_C$$

$$\Rightarrow T(f) = 11(1 + x^3) + (2x + 3x^2) - 4(2 + 5x^2 + x^3) + 6(x^2)$$

$$= 7x^3 - 11x^2 + 2x + 3$$

Problem 2 - 4 Points

Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 + 2x_3 \\ -x_1 + x_2 \\ 2x_2 + 5x_3 \end{bmatrix}$$

1. Write the standard matrix of the transformation.
2. Calculate the determinant of the standard matrix. Is the matrix invertible?

$$\underline{\underline{A}} = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 2 \\ 2 & 0 & 5 \end{pmatrix}$$

$$\det(\underline{\underline{A}}) = \begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & 2 \\ 2 & 0 & 5 \end{vmatrix} = \begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & 2 \\ 0 & 2 & 5 \end{vmatrix} = \begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{vmatrix}$$

$= 1$. So invertible.