Name: _____

Problem 1 - 6 Points

Fix bases

$$B = \{1, x, x^2\}, C = \{1 + x^3, 2x + 3x^2, 2 + 5x^2 + x^3, x^2\}$$

for $\mathbb{P}_2(\mathbb{R})$ and $\mathbb{P}_3(\mathbb{R})$, respectively. Let T be the linear transformation

$$T : \mathbb{P}_2(\mathbb{R}) \to \mathbb{P}_3(\mathbb{R})$$

with associated matrix

$$A_{B,C} = \begin{bmatrix} 3 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & -1 \\ 2 & 0 & 0 \end{bmatrix}$$

Calculate the polynomial $T(3 - 2x + 2x^2)$. You must give your answer as a polynomial.

Problem 2 - 4 Points

Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \right) = \begin{bmatrix} x_1 + 2x_3 \\ -x_1 + x_2 \\ 2x_2 + 5x_3 \end{bmatrix}$$

1. Write the standard matrix of the transformation.

2. Calculate the determinant of the standard matrix. Is the matrix invertible?