Quiz # 4 **Name:** Solutions
Date: 27/9/2022
Math 54: Fall 2022

Problem 1 - 2 Points

Consider the space $V = \mathbb{P}_2$, the polynomials with real coefficients of degree at most 2. Show whether the following subsets of V are subspaces of V.

1. $V_1 = \{a_0 + a_1 x + a_2 x^2, a_0, a_1, a_2 \text{ all odd}\}$
2. $V_2 = \{a_0 + a_1 x + a_2 x^2, a_0, a_1, a_2 \text{ all even}\}$

1) Not subspace. $0 \notin V_1$ as 0 is not odd.

2) Not subspace. e.g. $2+2x \in V_2$,

$\frac{1}{2}(2+2x) = 1+x \notin V_2$ not closed under scalar multiply

Problem 2 - 6 Points

True/False. Explain your answers.

1. If $B = \{\mathbf{v}_1, \mathbf{v}_2, ..., \mathbf{v}_n\}$ is a basis for $\mathbb{R}^n$ then the set $\{\mathbf{v}_1, ..., \mathbf{v}_n, \mathbf{u}\}$ must be linearly dependent for any vector $\mathbf{u}$.

True $\quad B$ basis

$\rightarrow$ exists coefficients $c_1, \dots, c_n$ s.t. $c_1\underline{v}_1 + \dots + c_n\underline{v}_n = \underline{u}$.

So $\{\underline{v}_1, \dots, \underline{v}_n, \underline{u}\}$ lin. dependent.

2. If the columns of an $n \times n$ matrix A form a basis for $\mathbb{R}^n$ then A is invertible and the columns of A^{-1} also form a basis for $\mathbb{R}^n$.

True $\quad \underline{A}^{-1}$ invertible, as $(\underline{A}^{-1})^{-1} = \underline{A}$,

so columns $\underline{A}^{-1}$ span $\mathbb{R}^n$ and are linearly independent.

3. If $B = \{f_1, ..., f_{n+1}\}$ is a basis for $\mathbb{P}_n$, the space of polynomials of degree at most n, then their derivatives $\{f'_1, ..., f'_{n+1}\}$ also forms a basis of $\mathbb{P}_n$.

False $\quad$ eg. $\{1, x\}$ span $\mathbb{P}_1$

$\downarrow$ differentiate

$\{0, 1\}$ does not span $\mathbb{P}_1$.

Problem 3 - 2 Points

Do one of the following:

1. Find the prime factors of 2149711.
2. Name a song you currently can't stop listening to. $\quad$ Fils de joie - Stromae
3. Draw something interesting.