Quiz # 3 **Name:** Solutions
Date: 15/9/2022
Math 54: Fall 2022

Problem 1 - 2 Points

Compute AB and BA for the following matrices. If the product is not defined explain why not.

$$A = \begin{bmatrix} 2 & 4 \\ 1 & -1 \\ -2 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 1 \\ 3 & -2 \end{bmatrix}$$

3×2 $\quad$ 2×2

$$\underline{\underline{A}}\,\underline{\underline{B}} = \begin{pmatrix} 2 & 4 \\ 1 & -1 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 3 & -2 \end{pmatrix} = \begin{pmatrix} 16 & -6 \\ -1 & 3 \\ -1 & -4 \end{pmatrix}$$

$\underset{2\times 2}{\underline{\underline{B}}}\,\underset{3\times 2}{\underline{\underline{A}}}$ not defined as # cols $B \neq$ # rows $\underline{\underline{A}}$

Problem 2 - 4 Points

Consider the following transformation $T : \mathbb{R}^3 \to \mathbb{R}^2$

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 \\ x_2 - x_3 \end{bmatrix}$$

Show whether this transformation is linear or not. If it is, find the standard matrix A of the transformation T.

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}\right) = T\left(\begin{bmatrix} x_1+y_1 \\ x_2+y_2 \\ x_3+y_3 \end{bmatrix}\right) = \begin{pmatrix} x_1 + y_1 \\ x_2+y_2 - x_3 - y_3 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 - x_3 \end{pmatrix} + \begin{pmatrix} y_1 \\ y_2 - y_3 \end{pmatrix}$$

$$= T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) + T\left(\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}\right)$$

$$T\left(c\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = T\left(\begin{bmatrix} cx_1 \\ cx_2 \\ cx_3 \end{bmatrix}\right) = \begin{pmatrix} cx_1 \\ cx_2 - cx_3 \end{pmatrix} = c\begin{pmatrix} x_1 \\ x_2 - x_3 \end{pmatrix} = cT\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right)$$

So T is linear.

$$T(\underline{e}_1) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad T(\underline{e}_2) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad T(\underline{e}_3) = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

$$\Rightarrow \underline{\underline{A}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \end{pmatrix}$$

Problem 3 - 4 Points

Suppose a linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ is defined by

$$T\left(\begin{bmatrix}1\\0\\0\end{bmatrix}\right) = \begin{bmatrix}1\\-1\\-1\end{bmatrix}, \quad T\left(\begin{bmatrix}0\\1\\0\end{bmatrix}\right) = \begin{bmatrix}0\\1\\1\end{bmatrix}, \quad T\left(\begin{bmatrix}0\\0\\1\end{bmatrix}\right) = \begin{bmatrix}0\\1\\0\end{bmatrix}$$

1. Write the standard matrix A of the transformation T. Find A^{-1}.
2. Compute $A^{-1}\begin{bmatrix}1\\-1\\-1\end{bmatrix}$. What do you get?

1) $\underline{\underline{A}} = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 1 \\ -1 & 1 & 0 \end{pmatrix}$, for $\underline{\underline{A}}^{-1}$,

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ -1 & 1 & 1 & 0 & 1 & 0 \\ -1 & 1 & 0 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & \\ 0 & 1 & 0 & \underbrace{\begin{matrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & -1 \end{matrix}}_{\underline{\underline{A}}^{-1}} \\ 0 & 0 & 1 & \end{array}\right)$$

2) $\underline{\underline{A}}^{-1}\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ as expected