Quiz # 2 **Name:** Solutions

Date: 8/9/2022

Math 54: Fall 2022

Problem 1 - 7 Points

Determine if $\begin{bmatrix} 3 \\ -5 \\ 8 \end{bmatrix}$ is in the span of $\begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$. If so write it as a linear combination of the two vectors.

$$x_1 \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix} + x_2 \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ -5 \\ 8 \end{pmatrix} \longleftrightarrow \begin{pmatrix} 1 & 2 \\ 3 & -1 \\ -2 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 3 \\ -5 \\ 8 \end{pmatrix}$$

$$\rightarrow \left(\begin{array}{cc|c} 1 & 2 & 3 \\ 3 & -1 & -5 \\ -2 & 3 & 8 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 2 & 3 \\ 0 & -7 & -14 \\ 0 & 7 & 14 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{array}\right)$$

$$\sim \left(\begin{array}{cc|c} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{array}\right) \Rightarrow -1 \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix} + 2 \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ -5 \\ 8 \end{pmatrix}$$

Problem 2 - 7 Points

Let the following vectors be given

$$\mathbf{v}_1 = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}, \ \mathbf{v}_2 = \begin{bmatrix} 3 \\ 4 \\ -1 \end{bmatrix}, \ \mathbf{v}_3 = \begin{bmatrix} -4 \\ h \\ 3 \end{bmatrix}$$

Find all values of h such that $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ do not span $\mathbb{R}^3$.

Do not span $\mathbb{R}^3$ $\Longleftrightarrow$ no pivot in every row of $\begin{pmatrix} \underline{v}_1 & \underline{v}_2 & \underline{v}_3 \end{pmatrix}$

$$\rightarrow \begin{pmatrix} 2 & 3 & -4 \\ 0 & 4 & h \\ 1 & -1 & 3 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & 3 \\ 0 & 4 & h \\ 2 & 3 & -4 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & 3 \\ 0 & 4 & h \\ 0 & 5 & -10 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & -1 & 3 \\ 0 & 1 & -2 \\ 0 & 4 & h \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & 3 \\ 0 & 1 & -2 \\ 0 & 0 & h+8 \end{pmatrix}$$

for no pivot, this is zero

$\Longrightarrow h=-8$, then does not span $\mathbb{R}^3$

Problem 3 - 6 Points

Determine whether the following statements are True/False. If True explain why, if False provide a counterexample.

1. A homogeneous system is always consistent.

True, $\underline{A}\,\underline{0} = \underline{0}$ always.

2. If S is a linearly dependent set of vectors, then each vector in S is a linear combination of the other vectors in S.

False. $\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$, $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ not in span $\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} \right\}$.

3. Columns of a 3×2 matrix can never span $\mathbb{R}^3$.

True. At most 2 pivots so no pivot in every row.