

Problem 1 - 2 Points

Write the following linear system as an augmented matrix

$$\begin{aligned}x_1 + x_2 + 3x_3 &= 3 \\2x_1 + 2x_2 - x_3 &= -1 \\x_1 + 3x_2 + 5x_3 &= -5\end{aligned}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 3 & 3 \\ 2 & 2 & -1 & -1 \\ 1 & 3 & 5 & -5 \end{array} \right)$$

Problem 2 - 4 Points

Row reduce the above augmented matrix to solve for the system. (Make sure to plug in the numbers at the end to make sure they work!)

$$\sim \left(\begin{array}{ccc|c} 1 & 1 & 3 & 3 \\ 0 & 0 & -7 & -7 \\ 0 & 2 & 2 & -8 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 3 & 3 \\ 0 & 1 & 1 & -4 \\ 0 & 0 & 1 & 1 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 3 & 3 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 1 \end{array} \right)$$

$$\sim \left(\begin{array}{ccc|c} 1 & 0 & 3 & 8 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 1 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 1 \end{array} \right)$$

$$x_1 = 5$$

$$x_2 = -5$$

$$x_3 = 1$$

Problem 3 - 4 Points

Consider the following augmented matrix

$$\begin{bmatrix} 1 & 2 & 4 & 3 \\ 0 & a & 1 & 6 \\ 0 & b & 2 & 4 \end{bmatrix}$$

Suppose that this augmented system is consistent. What can you say about a, b ?

$$\sim \left(\begin{array}{ccc|c} 1 & 2 & 4 & 3 \\ 0 & a & 1 & 6 \\ 0 & 0 & 2 - \frac{b}{a} & 4 - \frac{6b}{a} \end{array} \right), \text{ assuming } a \neq 0$$

\rightarrow If $\frac{6b}{a} = 4$, so $6b = 4a$, then consistent
as no pivot in last column

\rightarrow Otherwise, $2 - \frac{b}{a} \neq 0$, so $2a \neq b$

$$\text{If } a = 0, \sim \left(\begin{array}{ccc|c} 1 & 2 & 4 & 3 \\ 0 & b & 2 & 4 \\ 0 & 0 & 1 & 6 \end{array} \right), \text{ if } b \neq 0, \text{ then consistent.}$$

$$\text{If } b = 0, \sim \left(\begin{array}{ccc|c} 1 & 2 & 4 & 3 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 1 & 6 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & 2 & 4 & 3 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & 4 \end{array} \right),$$

inconsistent.

$\Rightarrow 2a \neq b$ then system consistent.