Quiz # 3 **Name:** ____________________________
Date: 15/9/2022
Math 54: Fall 2022

Problem 1a

Determine all values of k such that $\begin{bmatrix} 2 \\ 3 \\ k \end{bmatrix}$ is in the span $\left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix} \right\}$.

Problem 1b

Is $\left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix} \right\}$ a basis for $\mathbb{R}^3$?

Problem 2

Let $T : \mathbb{R}^3 \to \mathbb{R}^4$ be a linear transformation such that

$$T\left(\begin{bmatrix} 1 \\ -3 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} -2 \\ 3 \\ -4 \\ 0 \end{bmatrix},\ T\left(\begin{bmatrix} 0 \\ 3 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ -3 \\ 6 \\ 0 \end{bmatrix}, T\left(\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 3 \\ 2 \\ 0 \end{bmatrix} \}$$

1. Write the standard matrix A of the transformation T.
2. Is $A\mathbf{x} = \mathbf{b}$ consistent for any $\mathbf{b} \in \mathbb{R}^4$?
3. Find a basis for the column and null space of this transformation.
4. Verify the rank nullity theorem for this transformation.

Problem 3

For each of the following determine whether the statements are true or false. If true explain why. If false explain why not with a counterexample or otherwise.

1. Let A be invertible. Then A^T is invertible.

2. Let AB be invertible. Then both A and B are invertible.

3. There exists a linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^2$ such that $T\left(\begin{bmatrix}-1\\1\\-1\end{bmatrix}\right) = \begin{bmatrix}1\\0\end{bmatrix}$, $T\left(\begin{bmatrix}2\\-1\\0\end{bmatrix}\right) = \begin{bmatrix}-1\\1\end{bmatrix}$, $T\left(\begin{bmatrix}0\\-1\\2\end{bmatrix}\right) = \begin{bmatrix}0\\2\end{bmatrix}$.

4. Let $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ be a basis of $\mathbb{R}^3$ and $T : \mathbb{R}^3 \to \mathbb{R}^3$ be some linear transformation. Then $\{T(\mathbf{v}_1), T(\mathbf{v}_2), T(\mathbf{v}_3)\}$ is a basis of $\mathbb{R}^3$.