Welcome to Math 54! (Sections 215 | 218)

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HW is due on gradescope on **Tuesday 8pm**

- If not enrolled on gradescope, send me an email
- Please don’t email HW to me
- 1st HW due 30th August
- NO LATE SUBMISSIONS

Quizzes will be on Thursdays

- Quizzes 15 mins
- Lowest 2 scores dropped
- YOU CANNOT TAKE QUIZ FROM ANOTHER GSI
- NO MAKE-UP QUIZZES
- ALL QUIZZES IN PERSON
Linear Systems of Equations

**EQUATION**

\[ 2x + 3y = 4 \]
\[ 4x + y = -2 \]

**MATHEMATIC REPRESENTATION**

<table>
<thead>
<tr>
<th>left hand side</th>
<th>right hand side</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \begin{pmatrix} 2 &amp; 3 \ 4 &amp; 1 \end{pmatrix} ]</td>
<td>[ \begin{pmatrix} 4 \ -2 \end{pmatrix} ]</td>
</tr>
</tbody>
</table>

**Example:**

- Multiply top row \( \times 2 \)
  \[ 4x + 6y = 8 \]

- Subtract 1st eq. from 2nd
  \[ 5y = 10 \]
  \[ y = 2 \]

- Divide 2nd row \( \div 5 \)
  \[ 4x + y = -2 \]

- Subtract 1st row from 2nd
  \[ 4x = -4 \]
  \[ x = -1 \]
- Every linear equation can be written using a matrix representation
- Can perform the usual row operations on the matrix
- Linear Algebra is just the study of these equations and their matrices
Row operations

1) Swap rows

2) Multiply row by a scalar

3) Add a multiple of one row from another

These are called elementary row operations

The goal is to solve the linear equation using these operations
**Def.** Echelon Form

\[
\begin{pmatrix}
1 & 2 & 4 \\
0 & 2 & 1 \\
0 & 0 & 0 \\
\end{pmatrix}
\]

- Zero rows at bottom
- Leading non-zero entries in each row go from left to right
- Leading non-zero entries in each row are called pivots

Reduced Row Echelon Form

\[
\begin{pmatrix}
1 & 0 & 5 \\
0 & 1 & 10 \\
0 & 0 & 0 \\
\end{pmatrix}
\]

Echelon Form plus:

- Pivots are 1
- Anything above and below pivots are zero

**GOAL:** Use elementary row operations to reduce matrix to reduced row echelon form
**STRATEGY**

(Row 1 = \( R_1 \))

\[
\begin{align*}
\text{pivot} & \quad \begin{pmatrix} 1 & 2 & 1 & -1 \end{pmatrix} \\
\text{eliminate} & \quad \begin{pmatrix} 2 & 2 & 5 & 1 \end{pmatrix} \\
\text{eliminate} & \quad \begin{pmatrix} 3 & 5 & -2 & 5 \end{pmatrix}
\end{align*}
\]

**STEP 1:** Put into echelon form

- Find pivots and eliminate under pivot

\[
\begin{pmatrix} 1 & 2 & 1 & -1 \\
0 & -2 & 3 & 3 \\
0 & 0 & -1 & -8
\end{pmatrix}
\]

This is in echelon form.

**STEP 2:** Go from echelon to reduced row echelon.

- Make pivots 1

- Make entries above pivots 0
\[
\begin{pmatrix}
1 & 2 & 1 & -1 \\
0 & -2 & 3 & 3 \\
R_3 \div \frac{-3}{2} \rightarrow 0 & 0 & 1 & -1
\end{pmatrix}
\]

\[
R_1 - R_3 \rightarrow \begin{pmatrix}
1 & 2 & 0 & 0 \\
0 & -2 & 0 & 6 \\
0 & 0 & 1 & -1
\end{pmatrix}
\]

\[
R_2 - 3R_3 \rightarrow \begin{pmatrix}
1 & 2 & 0 & 0 \\
0 & -2 & 0 & 6 \\
0 & 0 & 1 & -1
\end{pmatrix}
\]

\[
R_2 \times \frac{-1}{2} \rightarrow \begin{pmatrix}
1 & 2 & 0 & 0 \\
0 & 1 & 0 & -3 \\
0 & 0 & 1 & -1
\end{pmatrix}
\]
This is now in reduced row echelon form!