

$$1.1) \quad f(x) = \ln\left(\frac{x+2}{x-1}\right)$$

$$\text{Need } \frac{x+2}{x-1} > 0,$$

Case 1: If  $x > 1$ , then  $x-1 > 0$ , so we need  $x+2 > 0$ , so  $x > -2$ .

Case 2: If  $x < 1$ , then  $x-1 < 0$ ,  
so need  $x+2 < 0$ , so  $x < -2$

$$\Rightarrow \text{Domain is } (-\infty, -2) \cup (1, \infty)$$

1.2) Yes. Because the function is one-to-one.

1.3) Horizontal asymptote at  $y=0$

$$\text{Because } \lim_{x \rightarrow \infty} f(x) = 0, \quad \lim_{x \rightarrow -\infty} f(x) = 0$$

Vertical asymptote at  $x=-2$  and  $x=1$ .

$$\text{Because } \lim_{x \rightarrow -2} f(x) = -\infty, \quad \lim_{x \rightarrow 1} f(x) = \infty.$$

$$2) \quad f(x) = 2^x - \frac{x^2}{4}$$

$$f(-2) = 2^{-2} - \frac{4}{4} = \frac{1}{4} - 1 = -0.75 < 0$$

$$f(-1) = 2^{-1} - \frac{1}{4} = \frac{1}{2} - \frac{1}{4} = \frac{1}{4} > 0.$$

$f$  is continuous between  $(-2, -1)$ .

So  $f(x) = 0$  for some  $-2 < x < -1$  by the IVT.

$$3.1) \quad \lim_{x \rightarrow \frac{\pi}{2}} \frac{100}{\tan(x)} \quad | \quad \lim_{x \rightarrow \frac{\pi}{2}^-} \tan(x) = +\infty \quad \lim_{x \rightarrow \frac{\pi}{2}^+} \tan(x) = -\infty.$$

$$\text{So } \lim_{x \rightarrow \frac{\pi}{2}^+} \frac{100}{\tan(x)} = \lim_{x \rightarrow \frac{\pi}{2}^-} \frac{100}{\tan(x)} = 0.$$

So the limit exists and is 0.

$$3.2) \quad \lim_{x \rightarrow 0} \operatorname{arcsin} \left( e^x - \frac{1}{2} \right), \quad e^0 - \frac{1}{2} = \frac{1}{2}$$

$\operatorname{arcsin}$  is continuous at  $\frac{1}{2}$ , so the limit equals

$$\operatorname{arcsin} \left( \lim_{x \rightarrow 0} e^x - \frac{1}{2} \right) = \operatorname{arcsin} \left( \frac{1}{2} \right) = \frac{\pi}{6}$$

$$4.1) \quad g(x) = f(2(x-1)) - 1$$

- Stretch horizontally by  $\frac{1}{2}$
- Shift right by 1
- Shift down by 1

4.2) Only need to check  $\lim_{x \rightarrow 1} f(x)$

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} \sqrt{2-2x} = 0$$

$$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} c+x = c+1$$

For continuity:  $\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^-} f(x) = f(1)$

$$\Rightarrow 0 = c+1, \quad \boxed{\text{so } c = -1.}$$