

Problem 1

1. Compute $\int x^n dx$. Treat the cases $n = -1, n \neq -1$ separately.
2. Compute $\int_{-3}^3 e^{-x^2} \sin(x) dx$. (Think about this in terms of the area definition of an integral)

$$1) \quad n = -1, \quad \int \frac{1}{x} dx = \ln(x) + C$$

$$n \neq -1, \quad \int x^n dx = \frac{1}{n+1} x^{n+1} + C$$

$$2) \quad e^{-x^2} \text{ is even.}$$

$$\sin(x) \text{ is odd.}$$

$$\Rightarrow e^{-x^2} \sin(x) \text{ is odd.}$$

$$\Rightarrow \int_{-3}^3 e^{-x^2} \sin(x) dx = 0.$$

Problem 2

A stone is dropped 450 metres above the ground. Note that acceleration due to gravity in normal person units is approximately 10 metres per second.²

1. How long does it take the stone to reach the ground?
2. With what velocity does it strike the ground ?

$$1) \quad \text{Acceleration: } x''(t) = -10 \text{ m/s}^2$$
$$\text{Velocity: } x'(t) = -10t + v_0 \text{ m/s}$$

Stone is dropped $\Rightarrow x'(0) = 0, \Rightarrow v_0 = 0.$

$$\text{Displacement: } x(t) = -5t^2 + s_0 \text{ m}$$

Initial displacement is 450m.

$$\Rightarrow x(0) = 450.$$

$$\text{Final time: } x(t^*) = 0 = -5t^{*2} + 450$$

$$(t^*)^2 = 90$$

$$\underline{\underline{t^* = 3\sqrt{10}}}$$

$$2) \quad \text{Final velocity} = x'(t^*) = \underline{\underline{-30\sqrt{10}}}.$$