Practice Quiz # 3 **Name:** Solutions
Date: 8/11/2021
Math 1A: Fall 2021

Problem 1

1. Compute the limit $\lim_{x\to\infty} \frac{\ln(x)}{x}$.
2. Compute the limit $\lim_{x\to\infty} x - \ln(x)$. (Try to find a way to use part 1)

1) $\frac{\ln(x)}{x} \xrightarrow{x\to\infty} \frac{\infty}{\infty}$, indeterminate

L'hôpital: $= \lim_{x\to\infty} \frac{\frac{1}{x}}{1} = 0$

2) $\lim_{x\to\infty} x - \ln(x) = \lim_{x\to\infty} x\left(1 - \frac{\ln(x)}{x}\right)$

$\lim_{x\to\infty} \left(1 - \frac{\ln(x)}{x}\right) = 1$ from part 1

$\Rightarrow \lim_{x\to\infty} \underbrace{x}_{\to\infty} \underbrace{\left(1 - \frac{\ln(x)}{x}\right)}_{\to 1} = \infty.$

Problem 2

Consider the function $f(x) = x^4 + x^2 - x$.

1. Show that the function has a critical point.
2. Show that the critical point of the function is unique.
3. Starting with $x_0 = \frac{1}{2}$, apply Newton's method to generate a better guess x_1 for the critical point of the function.

1) $f'(x) = 4x^3 + 2x - 1$, this is continuous

$f'(-10) = -4000 - 20 - 1 < 0$

$f'(10) = 4000 + 20 - 1 > 0$

$\Rightarrow$ For $f'(x) = 0$, a solution exists

2) $f''(x) = 12x^2 + 2$, $\Rightarrow f'(x)$ is differentiable

By Rolle's theorem (or MVT), as $f'(x)$ is differentiable, if $f'(x)$ has more than one root, $f''(x) = 0$ somewhere.

But $f''(x) = 12x^2 + 2 \geq 2$, so $f''(x) \neq 0$ implies unique critical point.

3) $x_1 = x_0 - \dfrac{f'(x_0)}{f''(x_0)}$, $x_0 = \frac{1}{2}$, NOTE: We are solving $f'(x) = 0$.

$\Rightarrow x_1 = \frac{1}{2} - \dfrac{\frac{4}{8} + 1 - 1}{12 \cdot \frac{1}{4} + 2} = \frac{1}{2} - \dfrac{1/2}{3+2} = \frac{1}{2} - \frac{1}{5} = \frac{4}{10} = \frac{2}{5}$.