Practice Quiz # 2 **Name:** Solutions
Date: 10/10/2021
Math 1A: Fall 2021

Problem 1

1. Rewrite a^x such that the base is in terms of e. (i.e. write it as $e^{f(x)}$ for some function f).
2. Differentiate this expression to show that the derivative of a^x is $\ln a \cdot a^x$.

1) $\log(a^x) = x\log(a)$, so $a^x = e^{\log(a^x)} = \underline{\underline{e^{x\log(a)}}}$

2) $\frac{d}{dx} a^x = \frac{d}{dx} e^{x\log(a)} = \log(a)\, e^{x\log(a)}$

$$= \underline{\underline{a^x \cdot \log(a)}}$$

Problem 2

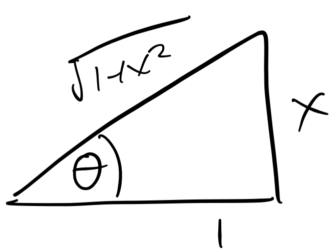
1. Fill in the blank $\tan(\tan^{-1}(x)) =$.
2. Find the derivative of $\tan^{-1}(x)$.
3. Using the chain rule, find the derivative of $\tan^{-1}(xe^x)$.

1) $\tan(\tan^{-1}(x)) = x$

2) Differentiating ↑ using chain rule

$$\Rightarrow \sec^2(\tan^{-1}(x)) \cdot \frac{d}{dx}\tan^{-1}(x) = 1$$

$$\frac{d}{dx}\tan^{-1}(x) = \cos^2(\tan^{-1}(x))$$



$\rightarrow \tan\theta = x$

$$\theta = \tan^{-1}(x)$$

$$\cos(\theta) = \cos(\tan^{-1}(x)) = \frac{1}{\sqrt{1+x^2}}$$

$$\Rightarrow \cos^2(\tan^{-1}(x)) = \frac{1}{1+x^2}$$

$$\Rightarrow \frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2}$$

3) Chain rule: $\frac{d}{dx}\tan^{-1}(xe^x) = \frac{1}{1+(xe^x)^2} \cdot (e^x + xe^x)$

$$= \frac{e^x(1+x)}{1+x^2e^{2x}}$$