

Problem 1

Find the derivative of the following function.

$$f(x) = \frac{x^2 - 2}{2x + 1}$$

$$\frac{d}{dx} \frac{u(x)}{v(x)} = \frac{vu' - uv'}{v^2} \quad , \quad u = x^2 - 2, \quad v = 2x + 1$$

$$u' = 2x, \quad v' = 2$$

$$\Rightarrow f'(x) = \frac{(2x+1)2x - (x^2-2)2}{(2x+1)^2}$$

$$= \frac{4x^2 + 2x - 2x^2 + 4}{(2x+1)^2}$$

$$= \frac{2x^2 + 2x + 4}{(2x+1)^2}$$

Problem 2

Find the equation of the line tangent to the curve $f(x) = \frac{x}{e^x}$ at $x = 0$.

$$f(0) = \frac{0}{1} = 0, \quad f(x) = xe^{-x}$$

$$f'(x) = e^{-x} - xe^{-x}$$

$$f'(0) = 1 - 0 = 1$$

$$\Rightarrow y = mx + b, \quad (x_1, y) = (0, 0), \quad m = 1$$

$$\Rightarrow 0 = 1 \cdot 0 + b, \quad b = 0$$

So line is $y = x$.