

### Problem 1

Consider the function  $f(x) = \sqrt{x} - \ln x$ , defined on the interval  $(0, \infty)$ .

1. On what interval(s) is  $f(x)$  increasing? Decreasing?
2. On what interval(s) is  $f(x)$  concave up? Concave down?
3. Find all local and global minima and maxima of  $f(x)$ .

### Problem 2

Is  $\sqrt{x} > \ln x$  for all  $x > 0$ ?

### Problem 3

Let  $g(x) = \sin^3(x)$  on the interval  $(-\pi, \pi)$ .

1. On what interval(s) is  $f(x)$  increasing? Decreasing? What are its critical numbers?
2. Determine whether each critical point is a local minimum, a local maximum, or neither.
3. Sketch a graph of  $f(x)$ .

### Problem 4

1. Find two positive numbers whose product is 100 and whose sum is a minimum.
2. A poster is to have an area of  $180 \text{ in}^2$  with 1-inch margins at the bottom and sides and a 2-inch margin at the top. What dimensions will give the largest printed area?

$$1.1) f(x) = \sqrt{x} - \ln(x), \quad x > 0$$

$$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} - \frac{1}{x}$$

$$\text{For increasing, } f'(x) > 0 \Rightarrow \frac{1}{2}x^{-\frac{1}{2}} - \frac{1}{x} > 0$$

$$\Rightarrow \frac{1}{2}\sqrt{x} > 1, \quad \sqrt{x} > 2, \quad x > 4.$$

$$\text{So } f'(x) > 0 \text{ for } x \in (4, \infty)$$

$$f'(x) = 0 \text{ for } x = 4$$

$$f'(x) < 0 \text{ for } x \in (0, 4)$$

$$1.2) f''(x) = -\frac{1}{4}x^{-\frac{3}{2}} + \frac{1}{x^2}$$

$$\text{Concave up: } f''(x) > 0, \quad -\frac{1}{4}x^{-\frac{3}{2}} + \frac{1}{x^2} > 0$$

$$\Rightarrow 1 > \frac{1}{4}\sqrt{x}, \quad 16 > x$$

$$\Rightarrow \text{Concave up: } x \in (0, 16), \text{ Concave down: } x \in (16, \infty)$$

$$1.3) \text{ Critical pts: } x=4 \text{ from part 1.1.}$$

$$f''(4) > 0 \text{ from part 1.2.}$$

$$\Rightarrow \text{Local min}$$

$$f(4) = 2 - \ln(4)$$

$$= 2(1 - \ln(2))$$

$$\text{Endpoints: } \lim_{x \rightarrow 0^+} f(x) = \infty$$

$$\lim_{x \rightarrow \infty} f(x) = \infty$$

$$\Rightarrow$$

$$\text{Global min at } x=4, \text{ (and local)}$$

$$\text{No local max}$$

$$\text{No global max}$$

2) If  $\sqrt{x} > \ln(x)$  for  $x > 0$ , then  $\sqrt{x} - \ln(x) > 0$  for  $x > 0$

$\Rightarrow$  Look at global min. of  $\sqrt{x} - \ln(x)$ .

We already found above that global min. was  $x=4$   
where  $f(4) = 2(1 - \ln(2))$ .

$\ln(2) < \ln(e) = 1$  as  $\ln(x)$  is an increasing fn.

$\Rightarrow f(4) > 0$ . So  $\sqrt{x} > \ln(x)$ ,  $x > 0$ .

3)  $g(x) = \sin^3(x)$ ,  $x \in (-\pi, \pi)$

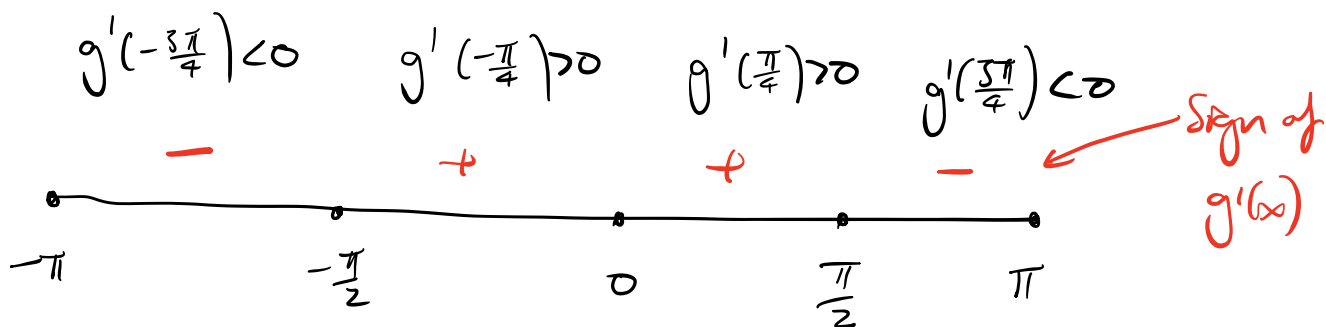
$$g'(x) = 3\sin^2(x)\cos(x)$$

$$g'(x) = 0 \Rightarrow \sin(x) = 0 \quad \text{or} \quad \cos(x) = 0$$

$$x = -\pi, 0, \pi$$

$$x = \pm \frac{\pi}{2}$$

a)



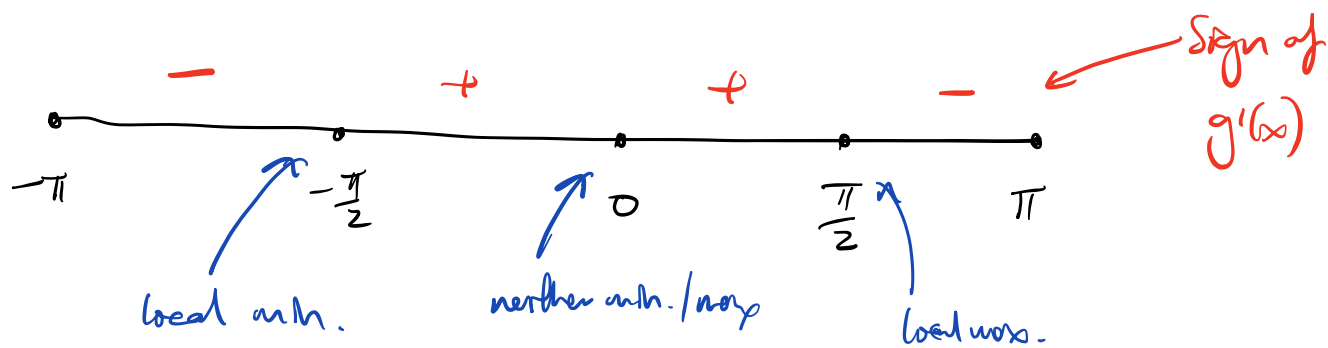
these are the only zeroes of  $g'(x)$ . Thus between these zeroes,  $g'(x)$  is either strictly  $> 0$  or  $< 0$  by continuity.

$\Rightarrow$  Check one point in each interval.

So decreasing  $(-\pi, -\frac{\pi}{2})$  and  $(\frac{\pi}{2}, \pi)$ .

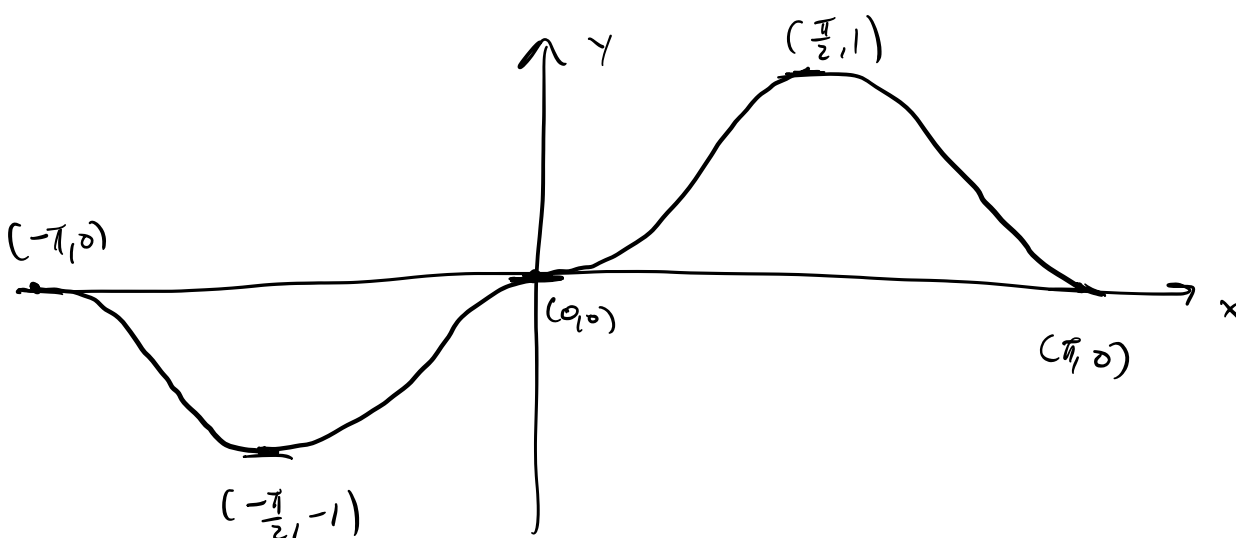
increasing  $(-\frac{\pi}{2}, 0)$  and  $(0, \frac{\pi}{2})$ .

b)



- Just check signs either side of critical point of  $g'(x)$ .

c)



4.1) Let  $x$  be one number. Then  $\frac{100}{x}$  is the other.

Want to minimize sum, so minimize  $x + \frac{100}{x}$

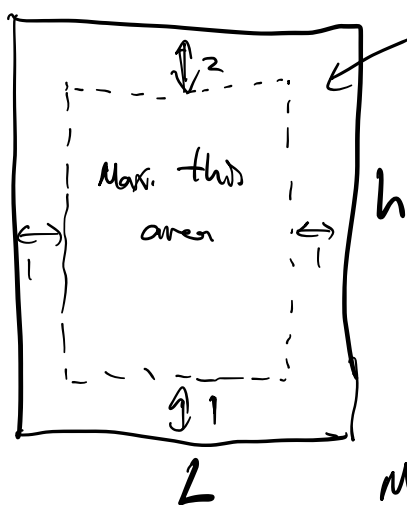
$$f(x) = x + \frac{100}{x}, \quad f'(x) = 1 - \frac{100}{x^2}$$

$$\text{Set } f'(x) = 0 \text{ to minimize, } 1 - \frac{100}{x^2} = 0 \quad x^2 = 100$$

$x = 10$ , want  $x$  positive.

$\Rightarrow$  the numbers are 10, 10.

4.2)



Total area is 180.

$$\text{Total Area: } 180 = hL$$

$$\text{Usable area: } (h-2) \cdot (L-2)$$

$$A(h) = (h-2) \left( \frac{180}{h} - 2 \right)$$

$$\text{Max. } A(h) \Rightarrow A'(h) = 0$$

$$A(h) = 180 - 2h - \frac{540}{h} + 6$$

$$A'(h) = -2 + \frac{540}{h^2} = 0$$

$$h^2 = 270$$

$$h = \sqrt{270} = \sqrt{9} \sqrt{30}$$

$$= 3\sqrt{30}$$

$$\Rightarrow L = \frac{180}{h} = \frac{180}{3\sqrt{30}} = \frac{60}{\sqrt{30}}$$