

### Problem 1

Find the linear approximation of the function  $f(x) = \sqrt{1-x}$ ,  $x = 0$  and use it to approximate the numbers  $\sqrt{0.9}$  and  $\sqrt{0.99}$ .

### Problem 2

Find the differential  $dy$  and evaluate  $dy$  for the given values of  $x$  and  $dx$

1.  $y = \cos \pi x$ ,  $x = \frac{1}{3}$ ,  $dx = -0.02$
2.  $y = \frac{x+1}{x-1}$ ,  $x = 2$ ,  $dx = 0.05$

### Problem 3

Use a linear approximation (or differentials) to estimate the given numbers

1.  $1/4.002$
2.  $e^{0.1}$

### Problem 4

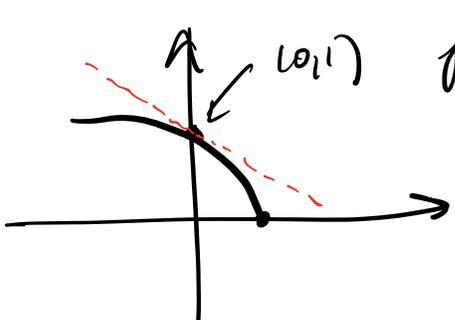
Find the domain of the following functions and find their critical points

1.  $f(x) = x^2 e^{-3x}$
2.  $g(t) = |3t - 4|$
3.  $h(p) = \frac{p^2}{p^2-4}$

### Problem 5

Find the critical points of the following on their domains

1.  $f(x) = x^{-2} \ln x$ ,  $x \in [\frac{1}{2}, 4]$
2.  $f(x) = x - 2 \tan^{-1}(x)$ ,  $x \in [0, 4]$
3.  $f(t) = t + \cot(\frac{t}{2})$ ,  $x \in [\frac{\pi}{4}, \frac{7\pi}{4}]$

1)   $f(x) = \sqrt{1-x}$ ,  $f'(x) = -\frac{1}{2\sqrt{1-x}}$ ,  $f'(0) = -\frac{1}{2}$

So straight line:  
 $y = -\frac{1}{2}x + c$ ,  $(x,y) = (0,1)$ ,  $1 = c$

$$\sqrt{0.9}, x=0.1 \Rightarrow \sqrt{0.9} \approx -\frac{1}{2} \cdot \frac{1}{10} + 1 = \frac{19}{20}$$

$$\sqrt{0.99}, x=0.01 \Rightarrow \sqrt{0.99} \approx -\frac{1}{2} \cdot \frac{1}{100} + 1 = \frac{199}{200}$$

2.1)  $y = \cos(\pi x)$ ,  $x = \frac{1}{3}$ ,  $dx = -0.02$

$$dy = -\pi \sin(\pi x) dx, dy = -\pi \sin\left(\frac{\pi}{3}\right) \cdot -0.02$$

$$= dy = \frac{\pi \cdot \frac{\sqrt{3}}{2} \cdot 2}{100} = \frac{\pi \sqrt{3}}{100}$$

2.2)  $y = \frac{x+1}{x-1}$ ,  $x = 2$ ,  $dx = 0.05$

$$dy = \frac{(x-1) - (x+1)}{(x-1)^2} dx = \frac{-2}{(x-1)^2} dx$$

$$dy = \frac{-2}{1^2} \cdot \frac{5}{100} = -\frac{1}{10}$$

$$3.1) \quad \frac{1}{4.002}, \text{ Linear approx. } f(x) = \frac{1}{x}$$

$$f(4) = \frac{1}{4}, \quad f(4.002) = \frac{1}{4.002}$$

use this

$$\rightarrow f'(x) = -\frac{1}{x^2}, \quad f'(4) = -\frac{1}{16}$$

$$\text{Linear approx. line: } y = -\frac{1}{16}x + C, \quad (x, y) = \left(4, \frac{1}{4}\right)$$

$$\frac{1}{4} = -\frac{1}{16} \cdot 4 + C, \quad C = \frac{1}{2}$$

$$\Rightarrow f(4.002) \approx -\frac{1}{16} \cdot 4.002 + \frac{1}{2} = 0.249875$$

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Differentials:

$$dy = -\frac{1}{x^2} dx, \quad dx = 0.002, \quad x = 4$$

$$\Rightarrow dy = -\frac{1}{16} \cdot 0.002$$

$$\Rightarrow f(4.002) \approx f(4) + dy = 0.25 - \frac{0.002}{16} = 0.249875.$$

$$3.2) \quad e^{0.1}, \quad f(x) = e^x$$

$$f(0) = 1, \quad f(0.1) = e^{0.1}$$

$$f'(x) = e^x, \quad f'(0) = 1$$

$$\Rightarrow \text{tangent approx. line} = y = x + c, \quad (x, y) = (0, 1)$$

$$\Rightarrow c = 1$$

$$\Rightarrow f(0.1) \approx 0.1 + 1 = 1.1$$

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$$\text{Differentials: } dy = e^x dx, \quad x=0, \quad dx=0.1$$

$$\Rightarrow dy = 0.1$$

$$\Rightarrow f(0.1) \approx f(0) + dy = 1.1$$

4.1)  $f(x) = x^2 e^{-3x}$ , Domain =  $\mathbb{R}$ , differentiable everywhere,

→ critical points:  $f'(x) = 0$

$$f'(x) = 2x e^{-3x} - 3x^2 e^{-3x} = 0$$

$$\Rightarrow 2x - 3x^2 = 0$$

$$x(2 - 3x) = 0$$

$\Rightarrow x = 0, x = \frac{2}{3}$  are critical points.

4.2)  $g(t) = |3t - 4|$ , Domain =  $\mathbb{R}$ ,

differentiable everywhere except at  $t = \frac{4}{3}$ .

$g'(t) = 0$  has no solutions.

$\Rightarrow t = \frac{4}{3}$  is critical point.

4.3)  $h(p) = \frac{p^2}{p^2 - 4} = \frac{p^2}{(p-2)(p+2)}$ , Domain =  $\mathbb{R} \setminus \{-2, 2\}$

Differentiable everywhere in domain.

$$h'(p) = 0 \Rightarrow p^2 = 0, p = 0.$$

$\Rightarrow$  Critical points:  $p = 0, p = -2, p = 2$

5.1)  $f(x) = \frac{\ln(x)}{x^2}$ , Differentiable everywhere in  $[\frac{1}{2}, 4]$

$$f'(x) = \frac{x^2 \cdot \frac{1}{x} - \ln(x) \cdot 2x}{x^4} = \frac{1 - 2\ln(x)}{x^3}$$

$$f'(x) = \frac{1 - 2\ln(x)}{x^3} = 0, \quad \frac{1}{2} = \ln(x), \quad x = e^{\frac{1}{2}}$$

$\Rightarrow$  Critical point at  $x = e^{\frac{1}{2}}$ .

5.2)  $f(x) = x - 2\arctan(x)$ ,  $x \in [0, 4]$

Differentiable everywhere in domain.

$$f'(x) = 1 - \frac{2}{1+x^2}, \quad f'(x) = 0 = \frac{1+x^2-2}{1+x^2}$$

$$\Rightarrow x^2 - 1 = 0, \quad x = 1, \quad x = -1$$

So critical points at  $x = 1$  only in domain.

$$5.3) f(t) = t + \cot\left(\frac{t}{2}\right), \quad t \in \left[\frac{\pi}{4}, \frac{7\pi}{4}\right]$$

Differentiable except at  $t = \pi$ .

$$f'(t) = 1 - \operatorname{cosec}^2\left(\frac{t}{2}\right) \cdot \frac{1}{2}$$

$$f'(t) = 0 = 1 - \frac{1}{2 \sin^2\left(\frac{t}{2}\right)}, \quad \sin^2\left(\frac{t}{2}\right) = \frac{1}{2}$$

$$\Rightarrow \sin\left(\frac{t}{2}\right) = \pm \frac{\sqrt{2}}{2}$$

$$\text{So } \frac{t}{2} = \frac{\pi}{4}, \quad t = \frac{\pi}{2}$$

$\Rightarrow$  Critical points at  $t = \frac{\pi}{2}, t = \pi$ .