Worksheet # 7 **Name:** Solutions
Date: 18/10/2021
Math 1A: Fall 2021

Problem 1

Log identities. For each of the expression simplify if possible. If no simplication is possible write None Possible.

1. $\ln(ab) = \cdot$ $\ln(a) + \ln(b)$
2. $\ln(a^b) = \cdot$ $b\ln(a)$
3. $\ln(a-b) = \cdot$ $\times$
4. $2^{\ln 2.5} = \cdot$ $\times$
5. $e^{\ln 2.5} = \cdot$ 2.5
6. $\frac{\ln(a)}{\ln(b)} = \cdot$ $\log_b(a)$

Problem 2

Simplify the following

1. $e^{\ln(\ln e^3)}$ $= e^{\ln(3\ln e)} = e^{\ln 3} = 3$
2. $\log_8 60 - \log_8 3 - \log_8$ [redacted] 2 $= \log_8\left(\frac{60}{3\cdot 2}\right) = \log_8(10)$
3. $\frac{1}{3}\ln(x+2)^3 + \frac{1}{2}\left(\ln x - \ln(x^2+3x+2)^2\right)$ $= \ln(x+2) + \ln\left(\frac{\sqrt{x}}{x^2+3x+2}\right) = \ln\left(\frac{\sqrt{x}}{x+1}\right)$
4. $8^{\frac{\ln 5}{\ln 2}}$ $= 8^{\log_2 5} = 2^{\log_2 5^3} = 5^3 = 125$

Problem 3

Differentiate the function $f(x) = \log_b(3x^2 - 2)$. (Hint: Find the derivative of $f(x) = b^x$ first.) For what value of b does $f'(1) = 3$?

Problem 4

Find $\lim_{x\to 0} \frac{\ln(1+x)}{x}$.

Problem 5

A population of protozoa develops with a constant relative growth rate of 0.7944 per member per day. On day zero the population consists of two members. Find the population after six days. (You do not need to use a calculator, leaving your answer as an expression is fine).

3) $y = \log_b(3x^2 - 2)$

$b^y = 3x^2 - 2$

$y' \cdot \ln(b)\, b^y = 6x$

$\Rightarrow y' = \dfrac{6x}{\ln(b)\, b^y} = \dfrac{6x}{\ln(b)(3x^2-2)}$

$y'(1) = \dfrac{6}{\ln(b)} = 3, \quad \ln(b) = 2, \quad b = e^2$

4) $\lim_{x \to 0} \dfrac{\ln(1+x)}{x} \to \dfrac{0}{0}$, so by L'hôpital's rule:

$= \lim_{x \to 0} \dfrac{\frac{1}{1+x}}{1} = 1$

5) Growth rate $= 0.7944$ per member

Population: $P(t) \Rightarrow P'(t) = 0.7944\, P(t)$

$\Rightarrow P(t) = Ce^{0.7944t}$

$P(0) = 2 = C$

$\Rightarrow P(6) = 2e^{6 \times 0.7944}$