Worksheet # 6 **Name:** Solutions
Date: 13/10/2021
Math 1A: Fall 2021

Problem 1

Differentiate the following

1. $f(x) = x\ln(x) - x$
2. $f(x) = \sin(\ln(x))$
3. $g(y) = \ln(ye^{-2y})$
4. $f(x) = \tan(\ln(ax + b))$

Problem 2

Find the following derivatives

1. $y = (x^2 + 2)^2(x^4 + 4)^4$
2. $y = \sqrt{\frac{x-1}{x^4+1}}$
3. $y = (\ln x)^{\cos(x)}$
4. $x^y = y^x$

Problem 3

1. Find $\frac{d}{dx^9}(x^8 \ln x)$
2. Find a formula for $f^{(n)}$ if $f(x) = \ln(x-1)$ where $f^{(n)}$ is the n-th derivative of f.

1.1) $f'(x) = \ln x + \frac{x}{x} - 1 = \ln x$

1.2) $f'(x) = \frac{1}{x}\cos(\ln(x))$

1.3) $f'(x) = \frac{1}{ye^{-2y}} \cdot \left(e^{-2y} - 2ye^{-2y}\right)$

$= \frac{1}{y}(1-2y)$

1.4) $f'(x) = \sec^2(\ln(ax+b)) \cdot \frac{1}{ax+b} \cdot a$

2.1) $\log y = 2\log(x^2+2) + 4\log(x^4+4)$

$\frac{y'}{y} = \frac{2}{x^2+2} \cdot 2x + \frac{4}{x^4+4} \cdot 4x^3$

$y' = \left(\frac{4x}{x^2+2} + \frac{16x^3}{x^4+4}\right) Y$

$= \left(\frac{4x}{x^2+2} + \frac{16x^3}{x^4+4}\right)\left((x^2+2)^2(x^4+4)^4\right)$

2.2) $\log(y) = \frac{1}{2}\left[\log(x-1) - \log(x^4+1)\right]$

$\frac{y'}{y} = \frac{1}{2}\left[\frac{1}{x-1} - \frac{4x^3}{x^4+1}\right]$

$y' = \frac{1}{2}\left[\frac{1}{x-1} - \frac{4x^3}{x^4+1}\right]\sqrt{\frac{x-1}{x^4+1}}$

2.3) $y = (\ln x)^{\cos(x)}$

$\log(y) = \cos(x)\log(\log(x))$

$\frac{y'}{y} = -\sin(x)\log(\log(x)) + \cos(x)\frac{1}{\log(x)}\cdot\frac{1}{x}$

$y' = \left(-\sin(x)\log(\log(x)) + \cos(x)\frac{1}{\log(x)}\cdot\frac{1}{x}\right)\ln(x)^{\cos(x)}$

2.4) $x^y = y^x$

$y\log(x) = x\log(y)$

$y'\log(x) + \frac{y}{x} = \log(y) + \frac{x}{y}\cdot y'$

$y'\left(\log(x) - \frac{x}{y}\right) = \log(y) - \frac{y}{x}$

$$y' = \frac{\log(y) - \frac{y}{x}}{\log(x) - \frac{x}{y}}$$

3.1) $\frac{d}{dx}\, x^8 \ln x = 8x^7 \ln x + \frac{x^8}{x}$

$= x^7(8\ln x + 1)$

$\frac{d^2}{dx^2}\, x^8 \ln x = \frac{d}{dx}\, x^7(8\ln x + 1)$

$= 7x^6(8\ln x + 1) \cdot x^7\left(\frac{8}{x}\right)$

$= x^6(8\cdot 7\cdot \ln x + 7 + 8)$

pick up a factor of n from x^n

$\frac{d}{dx}: x^7(8\ln x + 1)$ $\quad\Rightarrow\quad \frac{d^2}{dx^2}: x^6(8\cdot 7\ln x + 15)$

power x goes down by 1

some constant

Pattern:

$$\Rightarrow \frac{d^8}{dx^8}: x^0(8!\ln x + C)$$

$$\Rightarrow \frac{d^9}{dx^9}: \boxed{\frac{8!}{x}}$$

3.2) $f(x) = \ln(x-1)$

$$\frac{d}{dx}: \frac{1}{x-1}$$

$$\frac{d^2}{dx^2}: \frac{-1}{(x-1)^2}$$

$$\frac{d^3}{dx^3}: \frac{2}{(x-1)^3}$$

Pattern:

$$\Rightarrow \frac{d^n}{dx^n}: \frac{(-1)^{n-1}(n-1)!}{(x-1)^n}$$