

Problem 1

Find the following limits

1. $\lim_{x \rightarrow -\infty} \sqrt{4x^2 + 3x} + 2x$
2. $\lim_{x \rightarrow \infty} \sqrt{4x^2 + 3x} + 2x$

Problem 2

Find the equation of the line tangent to the curve at the given point of the following

1. $y = x^3 - 3x + 1$ at $x = 2$
2. $y = \frac{2x+1}{x+2}$ at $x = 0$

Problem 3

Evaluate the following derivatives:

1. $f(x) = 3x^2 - 4x + 1$
2. $f(t) = 2t^3 + t$
3. $f(t) = \frac{2t+1}{t+3}$
4. $f(x) = \frac{4}{\sqrt{1-x}}$

Problem 4

Evaluate derivatives using product/quotient rule:

1. $f(x) = \tan(x)$
2. $f(x) = \sec(x)$
3. $f(x) = \csc(x)$
4. (HARD) $f(x) = \arcsin(x)$

$$1) f(x) = \frac{(\sqrt{4x^2+3x} + 2x) \cdot (\sqrt{4x^2+3x} - 2x)}{\sqrt{4x^2+3x} - 2x} = \frac{3x}{\sqrt{4x^2+3x} - 2x} = \frac{3x \cdot \frac{1}{x}}{(\sqrt{4x^2+3x} - 2x) \cdot \frac{1}{x}}$$

$$\begin{aligned} 1.1) \lim_{x \rightarrow -\infty} f(x), x \text{ is negative. So } \frac{1}{x} = -\frac{1}{|x|^2} \cdot \lim_{x \rightarrow -\infty} f(x) &= \lim_{x \rightarrow -\infty} \frac{3}{\sqrt{\frac{1}{x^2} \sqrt{4x^2+3x}} - 2} \\ &= \lim_{x \rightarrow -\infty} \frac{3}{-\sqrt{4+\frac{3}{x}}} - 2 = \frac{3}{-4}. \end{aligned}$$

$$1.2) \lim_{x \rightarrow \infty} f(x), x \text{ is positive. So } \frac{1}{x} = \frac{1}{|x|^2} \cdot \lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow \infty} \frac{3}{\sqrt{\frac{1}{x^2} \sqrt{4x^2+3x}} - 2} = \lim_{x \rightarrow \infty} \frac{3}{\sqrt{4+\frac{3}{x}}} - 2 = \infty$$

$$2.1) f(x) = x^3 - 3x + 1$$

$$f(2) = 8 - 6 + 1 = 3, f'(x) = 3x^2 - 3, f'(2) = 12 - 3 = 9$$

So tangent line is $y = 9x + c$ ^{some number}

that passes through the point $(2, 3)$. Solve for c :

$$3 = 18 + c, \rightarrow c = -15$$

$$\Rightarrow \boxed{y = 9x - 15}$$

$$2.2) f(x) = \frac{2x+1}{x+2}, f(0) = \frac{1}{2}, f'(x) = \frac{(x+2) \cdot 2 - (2x+1)}{(x+2)^2}$$

$$f'(0) = \frac{2 \cdot 2 - 1}{2^2} = \frac{3}{4}$$

So tangent line is $y = \frac{3}{4}x + c$, passing through $(0, \frac{1}{2})$.

$$\Rightarrow \frac{1}{2} = c \quad , \quad \boxed{y = \frac{3}{4}x + \frac{1}{2}}$$

3.1) $f(x) = 3x^2 - 4x + 1$

$$f'(x) = 6x - 4$$

3.2) $f(x) = 2x^3 + 1$

$$f'(x) = 6x^2 + 1$$

3.3) $f(x) = \frac{2x+1}{x+3}$, Quotient Rule

$$f'(x) = \frac{(x+3)2 - (2x+1)}{(x+3)^2} = \frac{5}{(x+3)^2}$$

3.4) $f(x) = \frac{4}{\sqrt{1-x}} = 4(1-x)^{-\frac{1}{2}}$, use chain rule

$$f'(x) = -2 \cdot -1 \cdot (1-x)^{-\frac{3}{2}} = \frac{2}{(1-x)^{\frac{3}{2}}}$$

$$4.1) \quad f(x) = \tan(x) = \frac{\sin(x)}{\cos(x)}, \text{ Quotient Rule}$$

$$f'(x) = \frac{\cos(x) \cdot \cos(x) - \sin(x) \cdot -\sin(x)}{\cos^2(x)} = \frac{1}{\cos^2(x)} = \sec^2(x)$$

$$4.2) \quad f(x) = \sec(x) = \frac{1}{\cos(x)}$$

$$f'(x) = \frac{-1 \cdot -\sin(x)}{\cos^2(x)} = \frac{\sin(x)}{\cos(x)} \cdot \frac{1}{\cos(x)} = \sec(x) \tan(x)$$

$$4.3) \quad f(x) = \csc(x) = \frac{1}{\sin(x)}$$

$$f'(x) = \frac{-1 \cdot \cos(x)}{\sin^2(x)} = -\frac{1}{\sin(x)} \cdot \frac{\cos(x)}{\sin(x)} = -\csc(x) \cot(x)$$

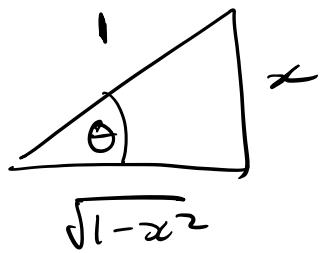
4.4) This question is meant to be hard. Don't worry too much about it for now, it was put as a challenge.

We know $\sin(\sin^{-1}(x)) = x$, take derivative both sides:

$$\frac{d}{dx} \sin^{-1}(x) \cdot \cos(\sin^{-1}(x)) = 1, \text{ Chain Rule}$$

$$\Rightarrow \frac{d}{dx} \sin^{-1}(x) = \frac{1}{\cos(\sin^{-1}(x))}$$

What is $\cos(\sin^{-1}(x))$?



$$\sin(\theta) = x$$

$$\theta = \arcsin(x)$$

$$\text{so } \cos(\theta) = \cos(\sin^{-1}(x))$$

$$= \sqrt{1-x^2}$$

$$\Rightarrow \frac{d}{dx} \sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$$