

Problem 1

Find the following infinite limits

1. $\lim_{x \rightarrow 5^+} \frac{x+1}{x-5}$

1) at $x=5$, $x-5=0$, so limit is $+\infty$ or $-\infty$.
 $x \rightarrow 5^+$, so $x > 5 \Rightarrow x+1 > 0$, $x-5 > 0$, so $\lim_{x \rightarrow 5^+} \frac{x+1}{x-5} = \infty$.

2. $\lim_{x \rightarrow \pi^-} \cot(x)$

2) At $x=\pi$, $\tan(x)=0$, so limit is $+\infty$ or $-\infty$.**Problem 2** $x \rightarrow \pi^-$, $\tan(x)$ is negative, so $\lim_{x \rightarrow \pi^-} \cot(x) = -\infty$.Given $\lim_{x \rightarrow 2} f(x) = 4$, $\lim_{x \rightarrow 2} g(x) = -2$, $\lim_{x \rightarrow 2} h(x) = 0$

Find the following limits if they exist. If not, explain why.

1. $\lim_{x \rightarrow 2} f(x) + 5g(x)$

1) $4 - 10 = -6$

2. $\lim_{x \rightarrow 2} \sqrt{f(x)}$

2) \sqrt{x} is continuous at $x=4$, so can move limit inside,

3. $\lim_{x \rightarrow 2} \frac{g(x)}{h(x)}$

$= \sqrt{\lim_{x \rightarrow 2} f(x)} = \sqrt{4} = 2$

Problem 33) $\lim_{x \rightarrow 2} h(x) = 0$, so we do not have enough info.

Evaluate the following limits:

1. $\lim_{t \rightarrow -2} \frac{t^4 - 2}{2t^2 - 3t + 2}$

1) Just plug in -2. $\rightarrow \frac{7}{8}$

2. $\lim_{x \rightarrow 4} \frac{x^2 + 3x}{x^2 - x - 12}$

2) $\frac{x^2 + 3x}{x^2 - x - 12} = \frac{x(x+3)}{(x+3)(x-4)}$ $\lim_{x \rightarrow 4^+} \frac{x}{x-4} = +\infty \Rightarrow \lim_{x \rightarrow 4^+} \frac{x}{x-4} \text{ DNE.}$
 $\lim_{x \rightarrow 4^-} \frac{x}{x-4} = -\infty$

Problem 4Show that $f(x)$ is continuous for all real numbers for the following:

1. $f(x) = \begin{cases} 1 - x^2, & x \leq 1 \\ \log(x), & x > 1 \end{cases}$

1) Only need to check $\lim_{x \rightarrow 1^-} f(x)$, $\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^-} f(x) = 0$
 $\lim_{x \rightarrow 1^-} f(x) = 0 = f(1)$. So $f(x)$ continuous everywhere.

2. $f(x) = \begin{cases} \sin(x), & x < \frac{\pi}{4} \\ \cos(x), & x \geq \frac{\pi}{4} \end{cases}$

2) only need to check $\lim_{x \rightarrow \frac{\pi}{4}^-} f(x)$, $\lim_{x \rightarrow \frac{\pi}{4}^+} f(x) = \lim_{x \rightarrow \frac{\pi}{4}^-} f(x) = \frac{\sqrt{2}}{2}$ So $\lim_{x \rightarrow \frac{\pi}{4}^-} f(x) = \frac{\sqrt{2}}{2} = f(\frac{\pi}{4}) \rightarrow f$ continuous everywhere.**Problem 5**Use the intermediate value theorem to show that the equation $e^x = 3 - 2x$ has a solution in the interval $(0, 1)$.

$f(x) = e^x - 3 + 2x$

f is continuous.

$f(0) = 1 - 3 = -2 < 0$

So $f(x) = 0$ for some $0 < x < 1$

$f(1) = e - 3 + 2 > 0$

by the IVT.

Problem 6

Find values for a, b such that the following function is continuous

$$f(x) = \begin{cases} \frac{x^2-4}{x-2}, & x < 2 \\ ax^2 - bx + 3, & 2 \leq x < 3 \\ 2x - a - b, & x \geq 3 \end{cases} \quad (1)$$

$$\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^-} \frac{(x-2)(x+2)}{x-2} = 4$$

$$\lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^+} ax^2 - bx + 3 = 4a - 2b$$

For continuity at $x=2$, need $\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^+} f(x)$

$$\Rightarrow \boxed{4a - 2b = 4}$$

$$\lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^-} ax^2 - bx + 3 = 9a - 3b$$

$$\lim_{x \rightarrow 3^+} f(x) = \lim_{x \rightarrow 3^+} 2x - a - b = 6 - a - b$$

For continuity at 3, need $\lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^+} f(x)$

$$\Rightarrow 9a - 3b = 6 - a - b$$

$$\boxed{10a - 2b = 6}$$

Combining the two boxed equations:

$$\Rightarrow \boxed{a = \frac{1}{3}, b = -\frac{4}{3}}$$