

### Problem 1

State the domain and range of each of the following functions and whether they are even/odd/neither:

1.  $f(x) = x$       Odd, Domain:  $\mathbb{R}$ , Range:  $\mathbb{R}$
2.  $f(x) = x^2$       Even, Domain:  $\mathbb{R}$ , Range:  $[0, \infty)$
3.  $f(x) = \sin(x)$       Odd, Domain:  $\mathbb{R}$ , Range:  $[-1, 1]$
4.  $f(x) = \frac{x^4 + x^2 - 3}{x^2 - 1}$       Even, Domain:  $x \neq -1, 1$ , Range:  $\mathbb{R}$
5.  $f(x) = \frac{x+1}{x^2-1} = \frac{1}{x-1}$       Neither, Domain:  $x \neq 1$ , Range:  $\mathbb{R}$

### Problem 2

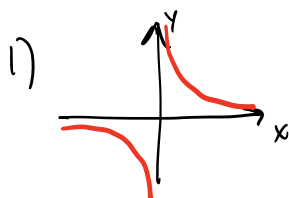
Sketch  $f(x) = x^2$ . Is it even or odd? Does this function have an inverse? Even, yes if define

$$f^{-1}(x) = \sqrt{x} \quad \text{OR} \quad f^{-1}(x) = -\sqrt{x} \quad \text{BUT NOT} \quad f^{-1}(x) = \pm\sqrt{x}.$$

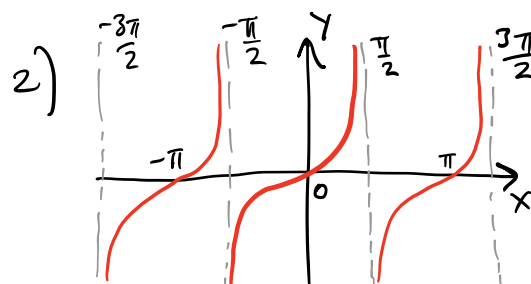
### Problem 3

Sketch each of the following, making sure to label where it crosses the x/y axes and say whether it is increasing/decreasing, odd/even, none of the above.

1.  $f(x) = \frac{1}{x}$
2.  $f(x) = \tan(x)$
3. (Hard)  $f(x) = \sin(\frac{1}{x})$



odd, neither increasing/decreasing



odd, neither increasing/decreasing

### Problem 4

The composition of two functions  $f(x), g(x)$  is a function  $h = f \circ g$  defined as  $h(x) = f(g(x))$ .

1. Is it the case that  $f \circ g = g \circ f$ ?
2. If  $f, g$  are odd, is  $f \circ g$  odd?
3. If  $f, g$  are even, is  $f \circ g$  even?

1) No. Consider  $f(x) = x^2, g(x) = \sin(x)$ .

2)  $f(g(-x)) = f(-g(x)) = -f(g(x))$ . Yes.

3)  $f(g(-x)) = f(g(x))$ . Yes.