Quiz # 5 **Name:** Solutions
Date: 13/11/2024
Math 128A - 106: Fall 2024

For full credit, please clearly show all your work.

Problem 1

Consider the ODE $y(t)' = -4y(t)$, with initial condition $y(0) = 3$.

1. *What is the analytical solution to this ODE?*
2. *Recall Euler's method $y(t+h) = y(t) + hy'(t)$. Apply one step of Euler's method to approximate the solution of $y(t)$ at $t = 0.25$.*

1) $y(t) = 3e^{-4t}$

2) $y(0.25) = y(0) + 0.25 \cdot y'(0)$

$y'(0) = -4y(0)$

$= 3 + \frac{1}{4} \cdot -4 \cdot 3$

$= 0$

Problem 2

Consider the Butcher table:

$$\begin{array}{c|cc} 0 & 0 & 0 \\ \frac{2}{3} & \frac{2}{3} & 0 \\ \hline & \frac{1}{4} & \frac{3}{4} \end{array}$$

1. *Write out the corresponding RK method. Is the method explicit or implicit? Why?*
2. *Apply one step of this RK method to approximate $y(t)$ at $t = 0.25$, where $y'(t) = -4y(t), y(0) = 3$. (Same ODE from the first question)*

1) $k_1 = f(t, y_n)$

$k_2 = f\left(t + \frac{2}{3}h, y_n + \frac{2}{3}k_1 \cdot h\right)$

$y_{n+1} = y_n + h\left[\frac{1}{4}k_1 + \frac{3}{4}k_2\right]$

2) $k_1 = -4 \cdot y_0 = -12$

$k_2 = -4\left[y_0 + \frac{2}{3} \cdot \frac{1}{4} \cdot -12\right] = -4 \cdot 1 = -4$

$\Rightarrow y(0.25) = 3 + \frac{1}{4}\left[\frac{1}{4} \cdot -12 + \frac{3}{4} \cdot -4\right]$

$= \frac{3}{2}$