Quiz # 4 **Name:** Solutions
Date: 30/10/2024
Math 128A - 105: Fall 2024

For full credit, please clearly show all your work.

Problem 1

Consider the quadrature rule for the domain $\int_0^1 f(x)\,dx$ *with points and weights given by:*

1. $x_1 = 0,\ x_2 = \frac{1}{6},\ x_3 = 1$
2. $w_1 = -\frac{1}{2},\ w_2 = \frac{6}{5},\ w_3 = \frac{3}{10}$

What is the precision of this quadrature rule?

Precision is 2

0: $\int_0^1 1\,dx = 1$

Quadrature: $w_1 + w_2 + w_3 = -\frac{1}{2} + \frac{6}{5} + \frac{3}{10} = 1$ ✓

1: $\int_0^1 x\,dx = \frac{1}{2}$, Quadrature: $\sum w_i x_i$

$= \frac{\cancel{6}}{5} \cdot \frac{1}{\cancel{6}} + \frac{3}{10} \cdot 1 = \frac{5}{10} = \frac{1}{2}$ ✓

2: $\int_0^1 x^2\,dx = \frac{1}{3}$, Quadrature: $\sum w_i x_i^2$

$= \frac{6}{5} \cdot \frac{1}{36} + \frac{3}{10} \cdot 1 = \frac{3}{10} + \frac{1}{30} = \frac{10}{30} = \frac{1}{3}$ ✓

3: $\int_0^1 x^3\,dx = \frac{1}{4}$, Quadrature $= \sum w_i x_i^3$

$= \frac{6}{5} \cdot \frac{1}{6^3} + \frac{3}{10} \cdot 1 = \frac{1}{5 \cdot 36} + \frac{3}{10} \neq \frac{1}{4}$ ✗

Problem 2

Use the quadrature rule from the previous question to approximate the integral

$$\int_1^4 e^x \, dx$$

You can leave your answer in terms of powers of e.

Change of variable:

$$y = \frac{x-1}{3}, \quad dy = \frac{1}{3}dx$$

$$\Rightarrow \int_0^1 e^{3y+1} \, dy \cdot 3$$

$$= 3\left[-\frac{1}{2}e^{3\cdot 0+1} + \frac{6}{5}e^{3\cdot\frac{1}{6}+1} + \frac{3}{10}e^{3\cdot 1+1}\right]$$

$$= 3\left[-\frac{1}{2}e + \frac{6}{5}e^{\frac{3}{2}} + \frac{3}{10}e^{4}\right]$$