Quiz # 3 **Name:** Solutions
Date: 9/10/2024
Math 128A - 105: Fall 2024

For full credit, please clearly show all your work.

Problem 1

Consider the function $f(x) = |x|$.

1. *Find a polynomial that interpolates the function at the points* $x = -2, 0, 1$ *using Lagrange interpolation.*
2. *What can you say about the error of the approximation using the error formula?*

a) $p(x) = \underbrace{f(-2)}_{=2} L_1(x) + \underbrace{f(0)}_{=0} L_2(x) + \underbrace{f(1)}_{=1} L_3(x)$

$L_1(x) = \frac{x-0}{-2-0} \cdot \frac{x-1}{-2-1} = \frac{1}{6} x(x-1)$

$L_3(x) = \frac{x+2}{1+2} \cdot \frac{x-0}{1-0} = \frac{1}{3} x(x+2)$

$\Rightarrow p(x) = \frac{2}{6} x(x-1) + \frac{1}{3} x(x+2)$

$= \frac{1}{3}\left[x(x-1) + x(x+2) \right] = \frac{1}{3}\left[2x^2 + x \right]$

b) Error formula: $|f(x) - p(x)| = \frac{f^{(n)}(\xi)}{n!} (\cdots)$

→ BUT, $f'(x)$ does not exist at $x=0$,

so can't be applied. → NOTHING can be said.

Problem 2

Find a polynomial $p(x)$ *using divided differences such that*

$$p(-1) = 1,\ p'(-1) = -6,\ p(3) = 9,\ p'(3) = 8$$

	$f(\cdot)$	$f(\cdot,\cdot)$	$f(\cdot,\cdot,\cdot)$	$f(\cdot,\cdot,\cdot,\cdot)$
-1	1	-6	$\frac{2--6}{4} = 2$	$\frac{-1/2}{4} = -\frac{1}{8}$
-1	1	$\frac{8}{4} = 2$	$\frac{8-2}{4} = \frac{3}{2}$	
3	9	8		
3	9			

$$\Rightarrow p(x) = 1 - 6(x+1) + 2(x+1)^2 - \frac{1}{8}(x+1)^2(x-3)$$