Quiz # 1 **Name:** Solutions
Date: 11/9/2024
Math 128A - 104: Fall 2024

For full credit please fully simplify your answers.

Problem 1

Consider the function $f(x) = 4x^3 - 3x^2 - \frac{5}{8}$ *on the interval* $x \in [0, 1]$.

1. *Find the maximum of the function on this domain.*
2. *Find the minimum of the function on this domain.*
3. *Does this function have a root* x_0 *where* $f(x_0) = 0$ *on this domain? Justify your answer.*

$f'(x) = 12x^2 - 6x$

Critical Pts: $f'(x) = 0$, $x = 0$, $x = \frac{1}{2}$

$f(0) = -\frac{5}{8}$, $f(\frac{1}{2}) = 4 \cdot \frac{1}{8} - 3 \cdot \frac{1}{4} - \frac{5}{8}$

$= \frac{4}{8} - \frac{6}{8} - \frac{5}{8} = -\frac{7}{8}$

Endpoints: $f(1) = 4 - 3 - \frac{5}{8} = \frac{3}{8}$

$\Rightarrow$ max: $(1, \frac{3}{8})$, min: $(\frac{1}{2}, -\frac{7}{8})$

$\rightarrow$ Root exists as sign changes between min and max of f on domain

Problem 2

Find the limit and rates of convergence of the following function as $h \to 0$:

$$f(h) = e^{h^2} + 2\cos(h)$$

$$e(x) = 1 + x + \frac{x^2}{2!} + \cdots$$

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots$$

$$e^{h^2} = 1 + h^2 + \frac{h^4}{2} + \cdots$$

$$2\cos(h) = 2 - h^2 + \frac{h^4}{12} - \cdots$$

$$\Rightarrow e^{h^2} + 2\cos(h) = 3 + h^2 - h^2 + \frac{h^4}{2} + \frac{h^4}{12} + \cdots$$

$$= 3 + \frac{7}{12}h^4 + \cdots$$

$\Rightarrow$ Limit is 3, convergence is $O(h^4)$