More polynomial interpolation

Error formula: $f(x) - p(x) = \dfrac{f^{(n)}(\xi)}{n!}(x-x_1)\cdots\cdots(x-x_n)$

Why? 1) $f^{(n)}(\xi)$, if $f(x)$ is polynomial of degree $n-1$, $p(x)$ should equal $f(x)$, so error should be 0

$$f^{(n)}(\xi) = 0$$

$\Longrightarrow f(x)$ is poly. of degree $n-1$

2) $(x-x_1)\cdots\cdots(x-x_n)$

We have $f(x_i) = p(x_i)$, so error at every x_i should be zero

3) $\dfrac{1}{n!}$, Taylor series term

Divided Differences

eg. Find polynomial through points

$(0,3)$, $(1,1)$, $(3,-1)$

→ DD table

x	$f(x_i)$	$f(x_1,x_2)$	$f(x_1,x_2,x_3)$
0	3	$\frac{1-3}{1-0} = -2$	
1	1		$\frac{1--2}{3} = 1$
3	3	$\frac{3-1}{3-1} = 1$	

list ascending order, much easier

$$\Rightarrow p(x) = 3 - 2(x-0) + 1(x-0)(x-1)$$

$$= 3 - 2x + x(x-1)$$

$$= 3 - 3x + x^2$$

Hermite Interpolation

Before: Want $p(x)$ s.t.

$$p(x_1) = f(x_1), \ldots, p(x_n) = f(x_n)$$

What if: Want $p(x)$ s.t.

$$p(x_1) = f(x_1), \quad p'(x_1) = f'(x_1)$$

$$p(x_2) = f(x_2), \quad p'(x_2) = f'(x_2) \text{ also?}$$

e.g. Find $p(x)$ s.t.

x	$f(x)$	$f'(x)$
0	1	2
1	-1	1

$\longrightarrow$ Divided differences

— With a slight tweak

x	$f(x)$	$f(x_1, x_2)$	$f(x_1, x_2, x_3)$	$f(x_1, x_2, x_3, x_4)$
0	1			
		$f'(0) = 2$		
0	1		$\frac{-2-2}{1-0} = -4$	
		$\frac{-1-1}{1-0} = -2$		$\frac{3--4}{1-0} = 7$
1	-1		$\frac{1--2}{1-0} = 3$	
		$f'(1) = 1$		
1	-1			

$$\Longrightarrow p(x) = 1 + 2(x-0) - 4(x-0)^2 + 7(x-0)^2(x-1)$$

$$= 1 + 2x - 4x^2 + 7x^2(x-1)$$

$$= 1 + 2x - 11x^2 + 7x^3$$