Gradient-Based Optimisation (10 Points)

Consider the problem of minimising the function $f(x) = x^2(x - 1)(x + 2)$ using gradient descent.

Problem 1 - 3 Points

Given a guess of the minimum of $f(x)$ at iteration $k$ denoted $x_k$, state the formula for the next guess $x_{k+1}$ using gradient descent with a rate of $\alpha$.

$$x_{k+1} = x_k - \alpha \frac{d}{dx} f(x_k)$$

Problem 2 - 4 Points

Sketch the function.

![Function Sketch](image)

Problem 3 - 4 Points

Let $x_0 = 0.5, \alpha = 0.1$. Do you expect gradient descent to converge to the absolute minimum of the function? Include a diagram in your explanation.

![Convergence Diagram](image)
Short Answer Questions (10 Points)

Problem 3 - 3 Points
State a possible ordering for depth-first search on the following graph:

```
   1
   |
  2-3
  |
  4-5
  |
  6-7
  |
  8
```

1, 2, 4, 5, 8, 6, 3, 7

Problem 6 - 2 Points
Write some mathematica code to plot the function \( f(x) = x^2(x - 1)(x + 2) \).

```
Plot[x^2 (x - 1) (x + 2), {x, -2, 2}]
```

Problem 7 - 2 Points
Write some mathematica code to plot the derivative of the function \( f(x) = x^2(x - 1)(x + 2) \).

```
Plot[D[x^2 (x - 1) (x + 2), x], {x, -2, 2}]
```

Problem 8 - 3 Points
What is your favourite and least favourite topics of this course? No wrong answers!

Hope you've enjoyed the semester!