Gradient-Based Optimisation (10 Points)

Consider the problem of minimising the function \( f(x) = x^2(x - 1)(x + 2) \) using gradient descent.

Problem 1 - 3 Points

Given a guess of the minimum of \( f(x) \) at iteration \( k \) denoted \( x_k \), state the formula for the next guess \( x_{k+1} \) using gradient descent with a fixed rate of \( \alpha \). (No need to differentiate the function explicitly you can just leave it as \( f'(x) \))

Problem 2 - 4 Points

Sketch the function.

Problem 3 - 4 Points

Let \( x_0 = 0.5, \alpha = 0.1 \). Do you expect gradient descent to converge to the absolute minimum of the function? Include a diagram in your explanation.
Short Answer Questions (10 Points)

Problem 3 - 3 Points
State a possible ordering for depth-first search on the following graph:

Problem 6 - 2 Points
Write some mathematica code to plot the function \( f(x) = x^2(x - 1)(x + 2) \) between \([-2, 2]\).

Problem 7 - 2 Points
Write some mathematica code to plot the derivative of the function \( f(x) = x^2(x - 1)(x + 2) \) between \([-2, 2]\).

Problem 8 - 3 Points
What is your favourite and least favourite topics of this course? No wrong answers!