Quiz # 5 **Name:** Solutions
Date: 13/4/2022
Math 124: Spring 2022

Numerical Solution of ODEs (10 Points)

Consider the ordinary differential equation $\frac{dy}{dt} = y, y(0) = y_0$.

Problem 1 (5 points)

Rewrite the differential equation using the fundamental theorem of calculus to obtain an expression for the solution to the equation $y(T)$ at some time $t = T$ in terms of y_0. (Hint: Your expression should include an integral $\int_0^T \cdot \, dt$).

$$y(T) = y_0 + \int_0^T y(t)\, dt$$

Problem 2 - 5 Points

Approximate the integral term in your expression above using a right Riemann sum. Rearrange your expression such that you have an explicit expression for $y(T)$ in terms of y_0. (Your answer here should involve no integrals).

$$y(T) = y_0 + \underbrace{\int_0^T y(t)\, dt}_{\text{Approximate this term } = T\,y(T)}$$

$$\Rightarrow y(T) = y_0 + T\,y(T)$$

$$y(T) = \frac{1}{1-T}\, y_0$$

This method is called Backward Euler. It is more reliable than the Forward Euler you were showed in class (if you are curious why feel free to come ask me.)

Object Orientation (10 Points)

The modulo of an integer n mod k is the remainder of n when divided by some integer base k. e.g. 13 mod 7 = 6, 8 mod 3 = 2.

Problem 3 - 5 Points

Define a struct called ModNumber that stores $(\ n \text{ mod } k\ )$. The constructor should take in two arguments ModNumber(n,k) and store the resulting value n mod k and the modulo base k.

```
struct ModNumber
    n::Int
    k::Int

    ModNumber(n, k) = new(n%k, k)
end
```

Problem 4 - 5 Points

Overload the addition operation for ModNumber such that it behaves as follows:

1. It is only defined for numbers with same modulo base, i.e. $(\ n_1 \text{ mod } k_1\ ) + (\ n_2 \text{ mod } k_2\ )$, $k_1 \neq k_2$ should error
2. $(\ n_1 \text{ mod k}\ ) + (\ n_2 \text{ mod } k\ ) = (\ n_1 + n_2\ ) \text{ mod k}$

```
function Base.:+(n1::ModNumber, n2::ModNumber)
    if n1.k != n2.k
        error("Can only sum ModNumber with same base.")
    end
    return (n1.n + n2.n) % n1.k
end
```