Quiz # 4 **Name:** Solutions
Date: 30/3/2022
Math 124: Spring 2022

Floating Point (10 Points)

The Float32 type in Julia represents numbers using 1 sign bit, 8 exponent bits, and 23 precision bits. Note that this representation is normalized. Explain your answers without using any code. No credit will be given to answers that are not properly justified.

Problem 1 (5 points)

What is the smallest Float32 ϵ such that $1 + \epsilon$ is a Float32 floating point number greater than 1?

1 in Float32: 0 | 001...1 | 0 - - - - - - - 0
sign | exponent | mantissa

(Binary) $= 1.0\ldots0 \times 2^0$, so next largest number is
23 places, as it is normalized

$\Rightarrow$ $1.0\ldots01 \times 2^0$, so $\epsilon = 2^{-23}$

Problem 2 - 5 Points

Is $2 + \epsilon$ a Float32 floating point number? (ϵ is the same as from the previous question)

2 in Float32: 0 | 010...0 | 0 - - - - 0
sign | exponent | mantissa

$= 1.0\ldots0 \times 2^1$, so exponent increases

So next largest number greater than 2 is

$= 1.0\ldots01 \times 2^1 = 2 + 2^{-22}$,

so $2+\epsilon$ is not a Float32 number.

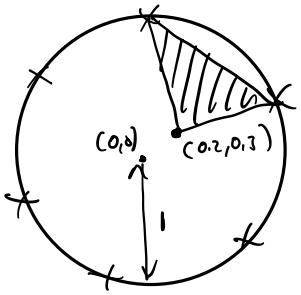
Computational Geometry + Runtime Analysis (10 Points)

Consider the following code

```
thetas = rand( n ) * 2 * pi
x = [ cos(t) for t in thetas]; y = [ sin(t) for t in thetas]
pt = [ 0.2, 0.3 ]
minarea = Inf; index = [ 0,0 ]
for i = 1:n
    for j = 1:n
        vec1 = [ x[i] - pt[1], y[i] - pt[2] ]
        vec2 = [ x[j] - pt[1], y[j] - pt[2] ]
        area = abs( vec1[1]*vec2[2] - vec1[2]*vec2[1] ) * 0.5
        if area < minarea
            minarea = area; index = [i,j];
        end
    end
end
```

Problem 3 - 5 Points

Describe briefly what the code does. You may find it helpful to draw a diagram.



Finds smallest triangle formed by connecting point at (0.2, 0.3) to two random points on the unit circle.

Problem 4 - 5 Points

What is the runtime in big O notation? What about the cost in memory?

Memory: $O(n)$

Runtime: $O(n^2)$