Matrix Multiply (20 Points)

In this problem you will implement matrix-matrix multiply two ways, using iteration and recursion. For all the problems we will assume that the matrices are of size \(2^k \times 2^k\) for some integer \(k \geq 0\). Recall also that matrix-matrix multiplication is defined as

\[
(AB)_{ij} = \sum_{k=1}^{n} A_{ik}B_{kj}
\]

Problem 1 (7 points)

Write a Julia function that takes in two input matrices \(A\) and \(B\) and returns the product \(C = AB\) using for loops. You may only do scalar multiplication and addition, and may not use any built in functionality that performs matrix-matrix or matrix-vector multiplication.

Now you will implement matrix-matrix multiply recursively. Do this by splitting the matrices as

\[
C = AB = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix} = \begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix}
\]

where \(A\) is size \(n \times n\) and each of the four submatrices is size \(\frac{n}{2} \times \frac{n}{2}\).
Problem 2 - 2 Points
What is the base case?

Problem 3 - 3 Points
Write out the expression for each submatrix $C_{ij}$ in terms of the submatrices $A_{ij}, B_{ij}$.

Problem 4 - 8 Points
Write Julia code to implement matrix-matrix multiply recursively using this splitting method.