

## Matrix Multiply (20 Points)

In this problem you will implement matrix-matrix multiply two ways, using iteration and recursion. For all the problems we will assume that the matrices are of size  $2^k \times 2^k$  for some integer  $k \geq 0$ . Recall also that matrix-matrix multiplication is defined as

$$(AB)_{ij} = \sum_{k=1}^n A_{ik} B_{kj}$$

### Problem 1 (7 points)

Write a Julia function that takes in two input matrices  $A$  and  $B$  and returns the product  $C = AB$  using for loops. You may only do scalar multiplication and addition, and may not use any built in functionality that performs matrix-matrix or matrix-vector multiplication.

Now you will implement matrix-matrix multiply recursively. Do this by splitting the matrices as

$$C = AB = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix} = \begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix}$$

where  $A$  is size  $n \times n$  and each of the four submatrices is size  $\frac{n}{2} \times \frac{n}{2}$ .

**Problem 2 - 2 Points**

What is the base case?

**Problem 3 - 3 Points**

Write out the expression for each submatrix  $C_{ij}$  in terms of the submatrices  $A_{ij}, B_{ij}$ .

**Problem 4 - 8 Points**

Write Julia code to implement matrix-matrix multiply recursively using this splitting method.