

Random Walk Paradox (20 Points)

In this problem we will investigate the random walk paradox. In 1D the problem can be stated as follows: Imagine an ant that walking on a 1D line that begins at 0 and at every timestep it can either move forwards or backwards one step with equal probability. How far away from 0 on average is the ant after n steps?

Problem 1 (5 points)

Write a Julia function that generates a random sequence of n integers each of which is either ± 1 that will signify at each step whether the ant moved forwards/backwards.
function generateRandomSequence(n)

sequence = [rand([-1,1]) for i=1:n]

return sequence
end

Problem 2 (3 points)

Given such a sequence, write some Julia code that calculates the displacement of the ant from zero after n steps.

displacement is signed distance from zero
displacement = sum(sequence)

Problem 3 - 6 Points

Using the functions from Problem 1 and 2 (you can receive full credit for this part even if your answers to Problems 1 and 2 are not entirely correct), simulate M trials of the random walk and find the average displacement of the ant from zero.

```
# Assume n is given
avg_disp = 0
for i = 1:M
    avg_disp += sum( generateRandomSequence(n) ) / M
end
println(avg_disp) # Print average displacement
```

The paradox is that in 1D/2D, as n gets large the average displacement tends to zero but in 3D it turns out this is no longer the case and the ant on average will drift off to infinity (surprisingly).

Problem 4 - 6 Points

Assume now that the ant moves forward with a probability of $P = 1 - 0.5 \cdot \exp\left(\frac{-x^2}{1000}\right)$ and backwards with probability $1 - P$ where x is its current location. Write a function that generates a random sequence of n integers ± 1 signifying the ant moving forwards/backwards with this adjustment.

```
func randDisp Biased (n)
    currPos = 0
    sequence = []
    for i = 1:n
        P = 1 - 0.5 * exp( -(x^2) / 1000 )
        if rand() < P
            push!(sequence, 1); currPos += 1
        else
            push!(sequence, -1); currPos -= 1
        end
    end
    return sequence
end
```