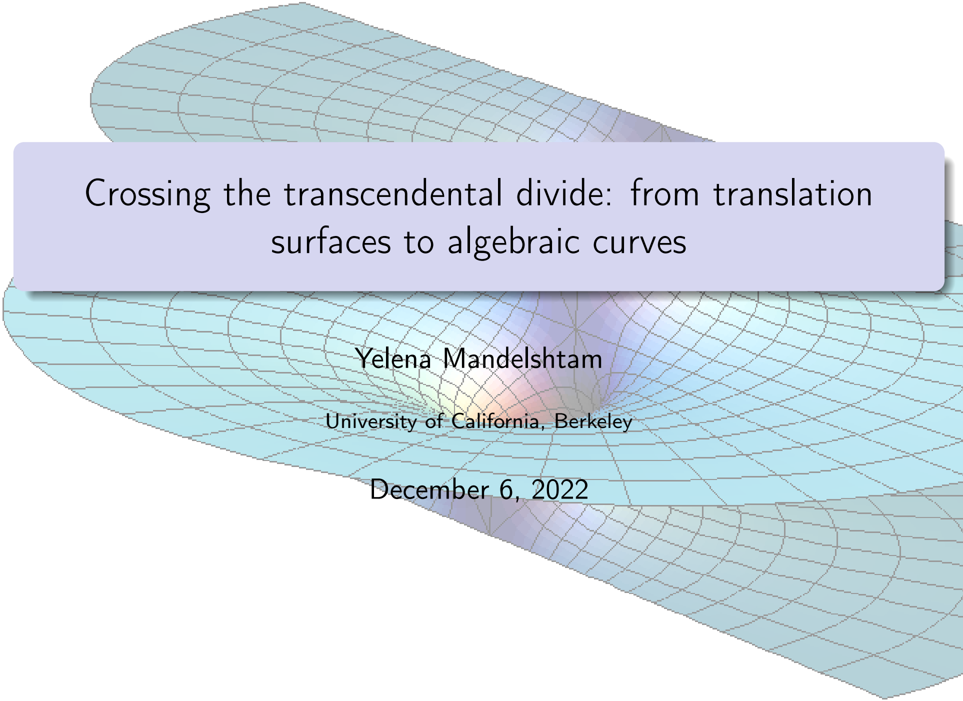

Crossing the transcendental divide: from translation surfaces to algebraic curves

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University of California, Berkeley

December 6, 2022

Work featured

- Crossing the transcendental divide: from translation surfaces to algebraic curves with Türkü Özlüm Çelik and Samantha Fairchild (arxiv 2211.00304)
    - Code can be found at https://mathrepo.mis.mpg.de/Tsurfaces2Acurves

Riemann Surfaces, analytic and algebraic

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Given a genus g surface X with homology basis $\alpha_1,\ldots,\alpha_g$ and $\beta_1,\ldots,\beta_g$. Let $\omega_1,\ldots,\omega_g$ be a basis of the space of differential forms on X. Then the *period matrix* of X is

$$\left(\begin{array}{ccc|ccc} \int_{\alpha_1}\omega_1 & \ldots & \int_{\alpha_g}\omega_1 & \int_{\beta_1}\omega_1 & \ldots & \int_{\beta_g}\omega_1 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \int_{\alpha_1}\omega_g & \ldots & \int_{\alpha_g}\omega_g & \int_{\beta_1}\omega_g & \ldots & \int_{\beta_g}\omega_g \end{array}\right)$$

Riemann matrices and theta functions

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If the period matrix of X is $(\tau_1|\tau_2) := \left(\left(\int_{\alpha_i}\omega_j\right)\middle|\left(\int_{\beta_i}\omega_j\right)\right)$, the matrix $\tau := \tau_1^{-1}\tau_2$ is called a *Riemann matrix* of X.

- $g \times g$ symmetric matrix with positive definite imaginary part, lies in the *Siegel upper half space* $\mathbb{H}_g$

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The theta function with characteristic $\varepsilon, \delta \in \{0,1\}^g$ is a complex-valued function defined on $\mathbb{C}^g \times \mathbb{H}_g$:

$$\theta\begin{bmatrix}\varepsilon\\ \delta\end{bmatrix}(\mathbf{z}|\tau) = \sum_{\mathbf{n}\in\mathbb{Z}^g} \exp\left(\pi i\left(\mathbf{n}+\frac{\varepsilon}{2}\right)^T\tau\left(\mathbf{n}+\frac{\varepsilon}{2}\right)+\left(\mathbf{n}+\frac{\varepsilon}{2}\right)^T\left(\mathbf{z}+\frac{\delta}{2}\right)\right).$$

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When $\varepsilon = \delta = 0$, this is the *Riemann theta function*

Theta functions and theta constants

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For fixed τ, the values $\theta\begin{bmatrix}\varepsilon\\ \delta\end{bmatrix}(\mathbf{0}|\tau)$ are known as *theta constants*. If $\varepsilon\cdot\delta\equiv 0,1 \pmod 2$ these are even, odd.

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- When $g = 4$ the Igusa modular form defines an analytic hypersurface (the Schottky locus) in terms of theta functions
- For higher genus, there are analytical equations in terms of theta functions defining a locus containing the Schottky locus
- the theta constants express certain divisors of the curve C, e.g. theta characteristic divisors, which recover the curve itself

Translation Surfaces

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A translation surface (X,ω) is given as the space obtained by identifying the sides of a collection of plane polygons pairwise by translations.

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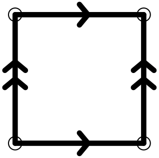
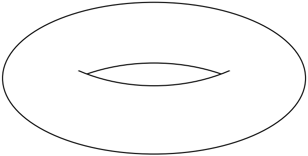
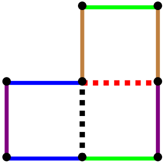
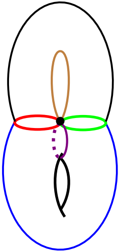
Examples:









Translation Surfaces are Riemann Surfaces

Equivalently, a translation surface is a Riemann surface together with a nonzero holomorphic one-form.

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The "Transcendental Divide"

It is well-known that every compact Riemann surface is an algebraic curve. How do we get the underlying algebraic curve *explicitly* starting from the translation surface?

Crossing the divide: past approaches

- find charts $\{(U,\psi_U) : U \text{ open in } X, \psi : U \to \mathbb{C}\}$ which satisfy the compatibility of charts to find the Riemann surface explicitly

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    - X comes automatically equipped with the one-form locally given by dz
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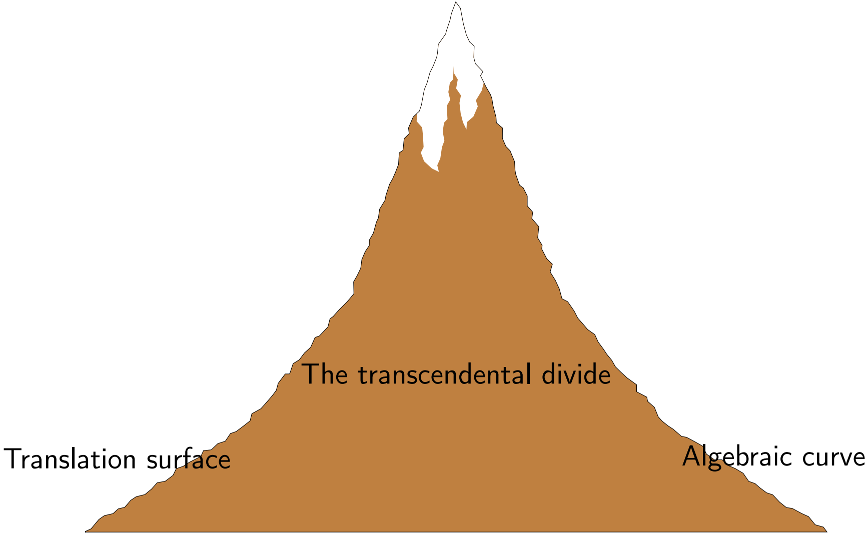
- Use odd theta characteristics to recover the curve.
  - hyperelliptic case: straightforward to find branch points using odd theta characteristics
  - non-hyperelliptic: has been done in low genera explicitly (odd theta characteristics define multitangent hyperplanes) by Lehavi $(g=4,5)$ and Çelik et.al. $(g=4)$
  - software packages for this exist in `Sage` and `magma`.

Our approach: through the Riemann matrix



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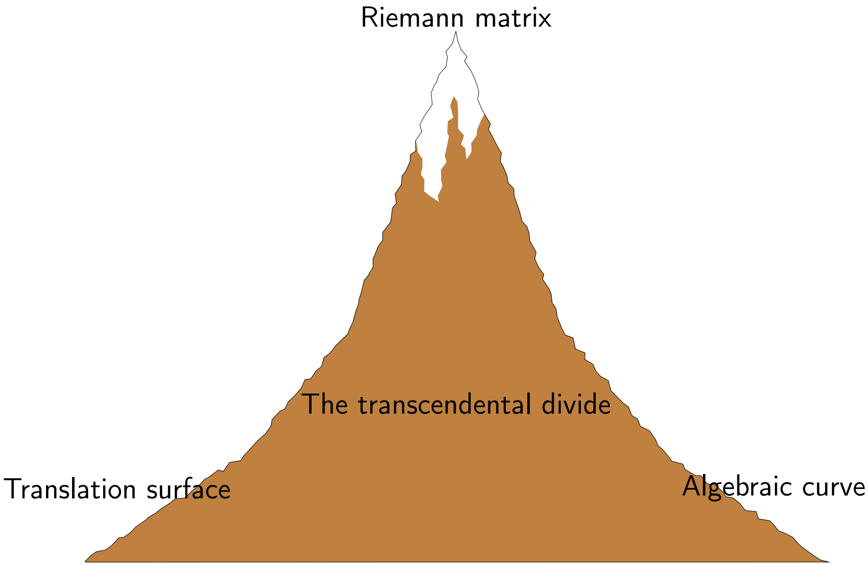

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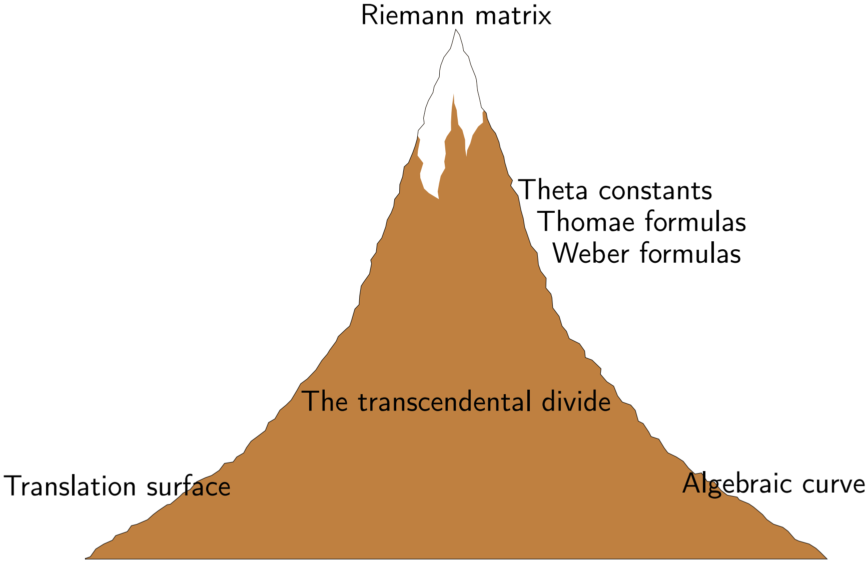

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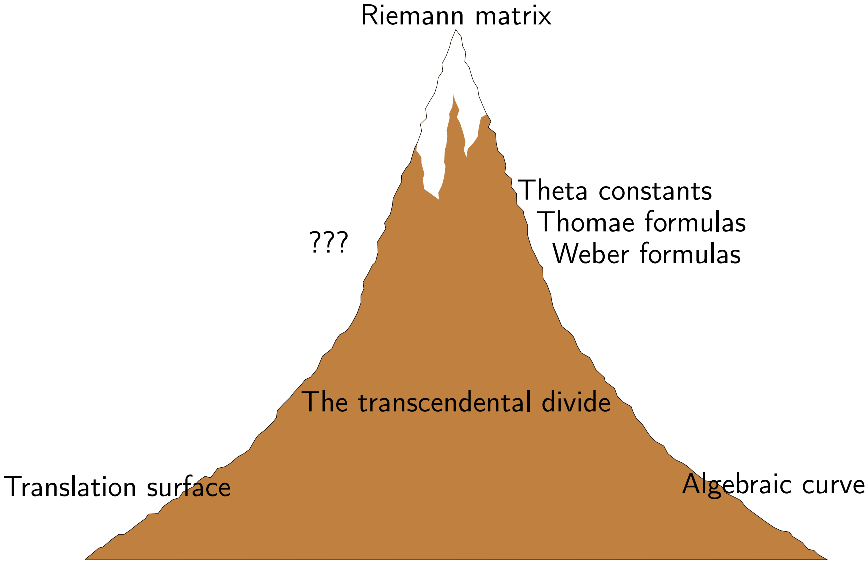
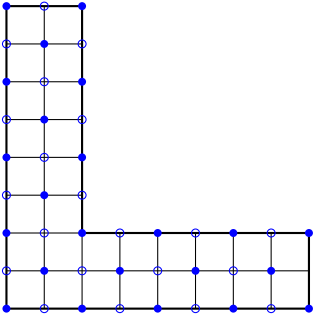


Our main tool: discrete Riemann surfaces

A *discrete Riemann surface* is a discrete analogue of a continuous Riemann surface.

- given by a cellular decomposition into Λ (black) and Λ^* (white) together with a discrete complex structure
- a discrete one-form ω is a complex function on the one-cells
- A symplectic basis $\alpha_1, \ldots, \alpha_g, \beta_1, \ldots, \beta_g$ becomes A and B-periods (by evaluating the integrals of ω along α_i and β_i)



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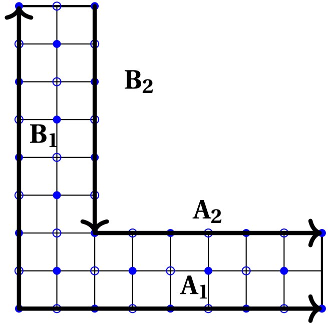

Discrete Riemann matrices

- $4g$ discrete black and white *periods* are defined as the integrals that are over the induced black and white closed paths
- there is a unique holomorphic differential such that the black and white A and B periods match a given set of $4g$ complex values (Bobenko and Günther, 2017)
- the *canonical basis* of holomorphic one-forms $\omega_1, \ldots, \omega_g$ is well defined where the black and white A periods are chosen to be equal with integration against the curves $\alpha_1, \ldots, \alpha_g$ to the identity matrix.
- The $g \times g$ *discrete period matrix* entries are the B periods with respect to the canonical basis.

Convergence results

Theorem (Bobenko-Skopenkov, 2016)

For a sequence of triangulations of a compact Riemann surface X with the maximal edge length tending to zero and with face angles bounded from zero, the discrete period matrices converge to a Riemann matrix of X.

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In this work, we consider square subdivisions rather than triangulations, per an announced result of Felix Günther.

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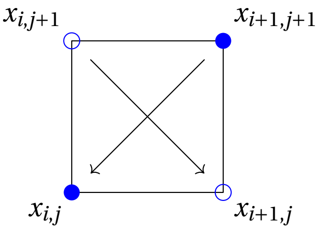

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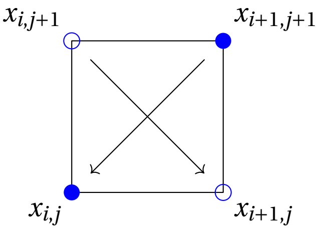

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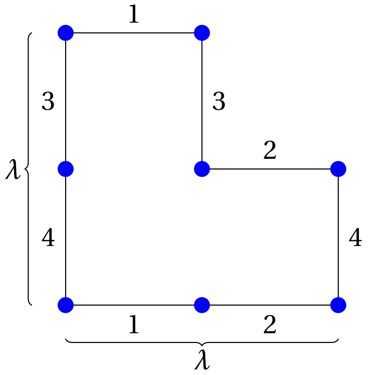
- From the presentation of our underlying surface as a translation surface, we get the periodicity equations. These essentially encode the side identifications

Our first candidate: the L-shape

Consider the family of translation surfaces given by the L-shape with one side length fixed to be 1 and the other (λ) varying



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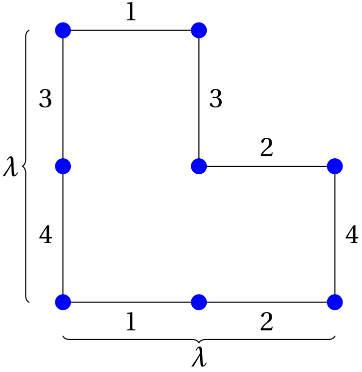

Theorem (Silhol, 2006 and Rodriguez, 2013)

The underlying Riemann matrix of the L-shape with side length λ is given by

$$\tau_\lambda = \frac{i}{2\lambda - 1}\begin{pmatrix} 2\lambda^2 - 2\lambda + 1 & -2\lambda(\lambda-1) \\ -2\lambda(\lambda-1) & 2\lambda^2 - 2\lambda + 1 \end{pmatrix}.$$

and the curve is given by

$$y^2 = x(x^2-1)(x-a)(x-1/a) \text{ for } a \neq -1, 0, 1$$

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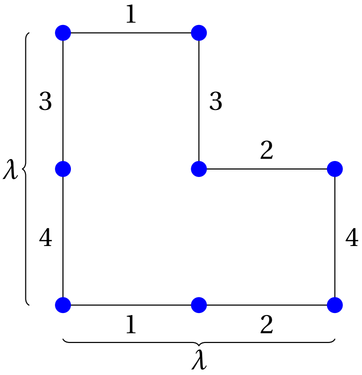

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Since the Riemann matrices and equations of the curves are already known for the L-shape, they are a perfect first candidate to test our procedure!

Our algorithm (L-shape)

Input: Let $\lambda = p/q$ be rational and reduced so that $\gcd(p,q) = 1$. Let $n \in \mathbb{N} \cup \{0\}$ be the level of subdivision.

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3. **Labelling vertices.** We label the vertices with the following bounds to match the L shape:

$$x_{i,j} = \left(\frac{i}{3^n s_\lambda}, \frac{j}{3^n s_\lambda}\right) \quad \text{for} \quad \begin{cases} 0 \le i \le 3^n s_\lambda & 0 \le j \le \lambda 3^n s_\lambda, \\ 3^n s_\lambda + 1 \le i \le \lambda 3^n s_\lambda & 0 \le j \le 3^n s_\lambda. \end{cases}$$

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5. **Periodicity equations.** We first choose a symplectic basis of the underlying Riemann surface. For all the A periods we have the following equations where the parity p is determined by $p = b$ if $i + j \equiv 0 \pmod 2$ and $p = w$ otherwise:

$$\begin{cases} 0 \le j \le 3^n s_\lambda & x_{\lambda 3^n s_\lambda, j} - x_{0,j} = A_1^p \\ 3^n s_\lambda \le j \le \lambda 3^n s_\lambda & x_{3^n s_\lambda, j} - x_{0,j} = A_1^p - A_2^p. \end{cases}$$

We compute similar equations for the B periods,

$$\begin{cases} 0 \le i \le 3^n s_\lambda & x_{i, \lambda 3^n s_\lambda} - x_{i,0} = B_1^p \\ 3^n s_\lambda \le i \le \lambda 3^n s_\lambda & x_{i, 3^n s_\lambda} - x_{i,0} = B_1^p + B_2^p. \end{cases}$$

In total we have $2(\lambda 3^n s_\lambda + 1)$ equations.

Our algorithm (L-shape)

6. **Final Normalizations.** In the final two normalizations we have the following number of equations:
   - 2 equations for normalization of holomorphic function ($x_{0,0} = x_{1,0} = 0$).
   - For each $k = 1,2$, there are 4 equations normalizing to the canonical basis. For $j = 1,2$:

$$A_j^w = A_j^b \quad \text{and} \quad A_j^w = \begin{cases} 1 & j = k \\ 0 & \text{else.} \end{cases}$$

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7. **Solving a system of equations for the discrete approximation.** For the kth row of the period matrix with $k = 1,2$, we obtain the equations by solving the system with a total of $9 + 3^{2n}(s_\lambda)^2(2\lambda - 1) + 2\lambda 3^n s_\lambda$ variables and $10 + 3^{2n}(s_\lambda)^2(2\lambda - 1) + 2\lambda 3^n s_\lambda$ equations. Each row is given by

$$\frac{1}{2}(B_1^w + B_1^b, B_2^w + B_2^b).$$

Example: $\lambda = 2$

n	Time	Approximation
0	0.02	$i\begin{pmatrix} 1.75 & -1.5 \\ -1.5 & 2.00 \end{pmatrix}$
1	0.05	$i\begin{pmatrix} 1.682276986822770 & -1.364553973645541 \\ -1.364553973645541 & 1.729107947291081 \end{pmatrix}$
2	0.37	$i\begin{pmatrix} 1.670169914926280 & -1.340339829852565 \\ -1.340339829852566 & 1.680679659705133 \end{pmatrix}$
3	3.92	$i\begin{pmatrix} 1.667472042082942 & -1.334944084165891 \\ -1.334944084165893 & 1.669888168331791 \end{pmatrix}$
4	28.23	$i\begin{pmatrix} 1.666852605322711 & -1.333705210645449 \\ -1.333705210645455 & 1.66741042129092 \end{pmatrix}$
5	255.10	$i\begin{pmatrix} 1.666709630962870 & -1.333419261925784 \\ -1.333419261925776 & 1.666838523851582 \end{pmatrix}$
6	2333.12	$i\begin{pmatrix} 1.666676596082551 & -1.333353192165260 \\ -1.33335319216523 & 1.666706384330567 \end{pmatrix}$
7	22786.59	$i\begin{pmatrix} 1.666668961530435 & -1.333337923061337 \\ -1.333337923061278 & 1.666675846122862 \end{pmatrix}$
∞	$i\begin{pmatrix} \frac{5}{3} & -\frac{4}{3} \\ -\frac{4}{3} & \frac{5}{3} \end{pmatrix}$	$i\begin{pmatrix} 1.66666666667 & -1.33333333333 \\ -1.33333333333 & 1.66666666667 \end{pmatrix}$

An irrational λ: one approach

Run a 0 level approximation for continued fraction approximations of λ. Here we take $\lambda = \frac{1+\sqrt{3}}{2}$.

Fraction	Tolerance	Time	0 level approximation
$\frac{15}{11}$	$1e-2$	0.33	$i\begin{pmatrix} 1.155267361944555 & -0.582252607292078 \\ -0.582252607292077 & 1.183447277345288 \end{pmatrix}$
$\frac{56}{41}$	$1e-3$	4.04	$i\begin{pmatrix} 1.15495696004714 & -0.578505984176023 \\ -0.57850598417602 & 1.159755674257178 \end{pmatrix}$
$\frac{209}{153}$	$1e-4$	56.96	$i\begin{pmatrix} 1.154756293461396 & -0.577572595239909 \\ -0.577572595239901 & 1.155583435806089 \end{pmatrix}$
$\frac{780}{571}$	$1e-5$	758.43	$i\begin{pmatrix} 1.154710996247692 & -0.577390320924590 \\ -0.577390320924568 & 1.154853829287965 \end{pmatrix}$
$\frac{780}{571}$	$1e-6$	758.43	$i\begin{pmatrix} 1.154710996247692 & -0.577390320924590 \\ -0.577390320924568 & 1.154853829287965 \end{pmatrix}$
$\frac{2911}{2131}$	$1e-7$	744.47	$i\begin{pmatrix} 1.15471099624769 & -0.577390320924590 \\ -0.577390320924568 & 1.154853829287965 \end{pmatrix}$
$\frac{10864}{7953}$	$1e-8$	774.37	$i\begin{pmatrix} 1.154710996247692 & -0.57739032092459 \\ -0.577390320924568 & 1.154853829287965 \end{pmatrix}$
$\frac{40545}{29681}$	$1e-9$	871.63	$i\begin{pmatrix} 1.154710996247692 & -0.577390320924590 \\ -0.577390320924568 & 1.154853829287965 \end{pmatrix}$
	Exact:	$\frac{i}{\sqrt{3}}\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$	$i\begin{pmatrix} 1.15470053838 & -0.57735026919 \\ -0.57735026919 & 1.15470053838 \end{pmatrix}$

An irrational λ: another approach

Fix some continued fraction approximation (here, when $\lambda = \frac{1+\sqrt{3}}{2}$, we take $\frac{10864}{7953}$) and run further levels of subdivision.

Level	Time	Approximation
0	774.37	$i\begin{pmatrix} 1.154710996247692 & -0.57739032092459 \\ -0.577390320924568 & 1.154853829287965 \end{pmatrix}$
1	7290.33	$i\begin{pmatrix} 1.154702501426855 & -0.577358617765855 \\ -0.577358617765802 & 1.154735511279386 \end{pmatrix}$
2	72388.83	$i\begin{pmatrix} 1.154700538230285 & -0.577351291004541 \\ -0.577351291004404 & 1.154708167385750 \end{pmatrix}$
∞	$\frac{10864}{7953}$	$i\begin{pmatrix} 1.15470053534 & -0.5773502631 \\ -0.5773502631 & 1.15470053534 \end{pmatrix}$
∞	$\frac{1+\sqrt{3}}{2}$	$i\begin{pmatrix} 1.15470053838 & -0.57735026919 \\ -0.57735026919 & 1.15470053838 \end{pmatrix}$

The Jenkins-Strebel representatives

The JS differentials are a family of surfaces defined for any genus

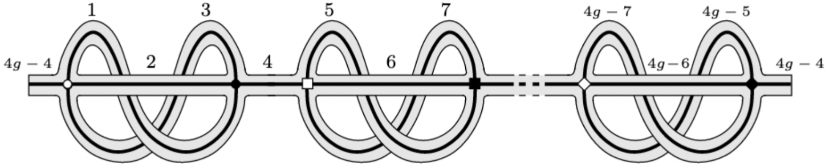
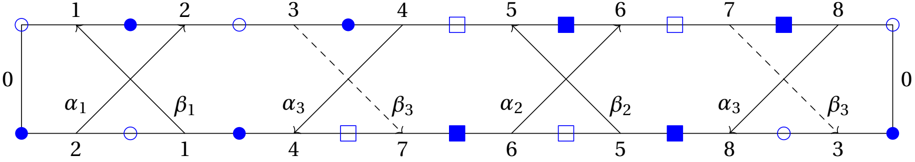


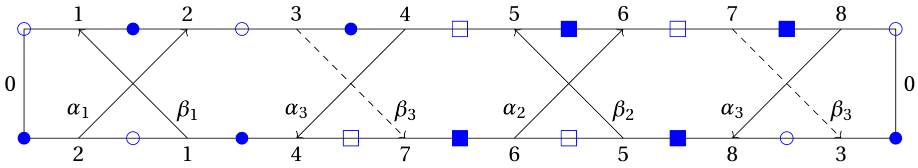
Figure: The *ribbon graph* associated to a JS differential of genus g

These can also be drawn as a $1 \times (4g-4)$ rectangle:



Our algorithm (JS surfaces)

The same as for the L-shapes! Except getting the periodicity relations is MUCH harder :(



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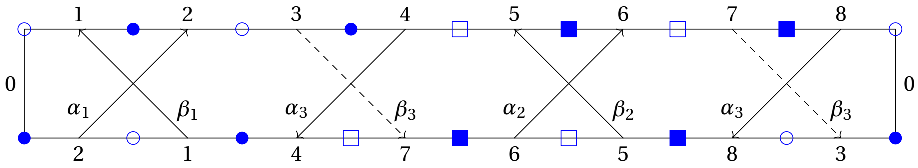
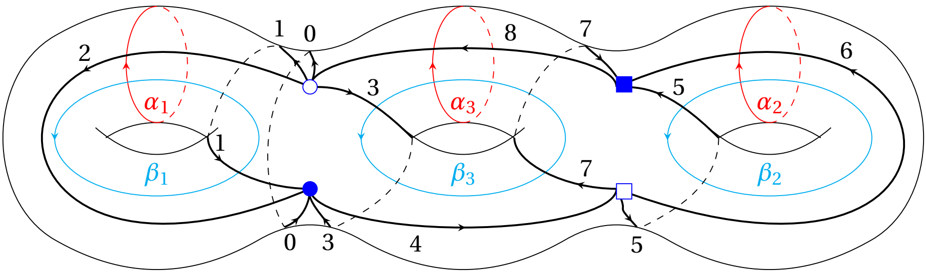
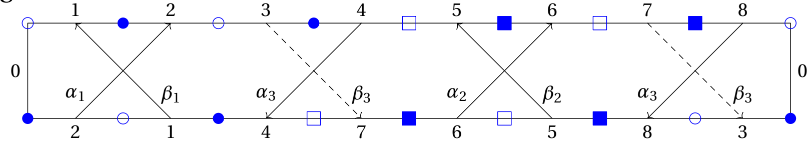
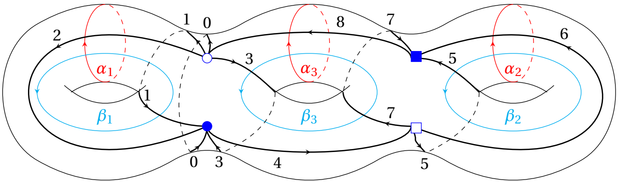




Periodicity relations on the JS surfaces

For $g = 3$ we have





This leads to relations like

$$3_{top} - 3_{bottom} = -\beta_3 + 7_{bottom} - 7_{top} = -\beta_3 + \alpha_3 - \alpha_2 + \beta_2.$$

And in higher genus

$$3_{top} - 3_{bottom} = -\beta_g + (g-2)\alpha_g + \sum_{j=2}^{g-1} (\beta_j - \alpha_j)$$

genus 2

n	Time	Approximation
0	0.01	$\begin{pmatrix} i & 0 \\ 0 & i \end{pmatrix}$
1	0.01	$\begin{pmatrix} -0.162162162162162+0.972972972972973i & 0.162162162162162+0.027027027027027i \\ 0.162162162162162+0.027027027027027i & -0.162162162162162+0.972972972972973i \end{pmatrix}$
2	0.01	$\begin{pmatrix} -0.181145110935355+0.966032669224184i & 0.181145110935355+0.033967330775816i \\ 0.181145110935355+0.033967330775816i & -0.181145110935355+0.966032669224184i \end{pmatrix}$
3	0.88	$\begin{pmatrix} -0.183154151609459+0.965246769734320i & 0.183154151609458+0.034753230265680i \\ 0.183154151609459+0.034753230265680i & -0.183154151609458+0.965246769734320i \end{pmatrix}$
4	6.07	$\begin{pmatrix} -0.183376430458456+0.965159203662913i & 0.183376430458456+0.034840796337087i \\ 0.183376430458456+0.034840796337087i & -0.183376430458456+0.965159203662913i \end{pmatrix}$
5	61.02	$\begin{pmatrix} -0.183401116934991+0.965149470930579i & 0.183401116934991+0.034850529069421i \\ 0.183401116934991+0.034850529069421i & -0.183401116934991+0.965149470930579i \end{pmatrix}$
6	539.43	$\begin{pmatrix} -0.183403859739525+0.965148389476567i & 0.183403859739525+0.034851610523432i \\ 0.183403859739525+0.034851610523432i & -0.183403859739525+0.965148389476568i \end{pmatrix}$
7	3969.65	$\begin{pmatrix} -0.183404164493890+0.965148269314525i & 0.183404164493890+0.034851730685474i \\ 0.183404164493890+0.034851730685475i & -0.183404164493890+0.965148269314525i \end{pmatrix}$

genus 2

n	Time	Approximation
0	0.01	$\begin{pmatrix} i & 0 \\ 0 & i \end{pmatrix}$
1	0.01	$\begin{pmatrix} -0.162162162162162 + 0.972972972972973i & 0.162162162162162 + 0.027027027027027i \\ 0.162162162162162 + 0.027027027027027i & -0.162162162162162 + 0.972972972972973i \end{pmatrix}$
2	0.01	$\begin{pmatrix} -0.181145110935355 + 0.966032669224184i & 0.181145110935355 + 0.033967330775816i \\ 0.181145110935355 + 0.033967330775816i & -0.181145110935355 + 0.966032669224184i \end{pmatrix}$
3	0.88	$\begin{pmatrix} -0.183154151609459 + 0.965246769734320i & 0.183154151609458 + 0.034753230265680i \\ 0.183154151609459 + 0.034753230265680i & -0.183154151609458 + 0.965246769734320i \end{pmatrix}$
4	6.07	$\begin{pmatrix} -0.183376430458456 + 0.965159203662913i & 0.183376430458456 + 0.034840796337087i \\ 0.183376430458456 + 0.034840796337087i & -0.183376430458456 + 0.965159203662913i \end{pmatrix}$
5	61.02	$\begin{pmatrix} -0.183401116934991 + 0.965149470930579i & 0.183401116934991 + 0.034850529069421i \\ 0.183401116934991 + 0.034850529069421i & -0.183401116934991 + 0.965149470930579i \end{pmatrix}$
6	539.43	$\begin{pmatrix} -0.183403859739525 + 0.965148389476567i & 0.183403859739525 + 0.034851610523432i \\ 0.183403859739525 + 0.034851610523432i & -0.183403859739525 + 0.965148389476568i \end{pmatrix}$
7	3969.65	$\begin{pmatrix} -0.183404164493890 + 0.965148269314525i & 0.183404164493890 + 0.034851730685474i \\ 0.183404164493890 + 0.034851730685475i & -0.183404164493890 + 0.965148269314525i \end{pmatrix}$

Now we can use the Riemann matrix from level 7 to evaluate the theta constants and find the equation of the underlying curve!

Genus 2 experiments

- Using the Sage package `RiemannTheta` we find the odd theta constants which give us the six branch points of the hyperelliptic curve:

$$\alpha_1 := -3.55001177927944 + i \cdot 9.27369555271397,$$
$$\alpha_2 := -0.0360027110167584 - i \cdot 0.0940498797751955,$$
$$\alpha_3 := 0.603906137193071 + i \cdot 3.24517640725254,$$
$$\alpha_4 := 0.0554252204362169 - i \cdot 0.297835386410189,$$
$$\alpha_5 := 3.90800485599692 - i \cdot 7.79154768793860,$$
$$\alpha_6 := 0.0514341663705383 + i \cdot 0.102546382318450.$$

- Observe that $\alpha_1 \cdot \alpha_2 = \alpha_3 \cdot \alpha_4 = \alpha_5 \cdot \alpha_6 = 1$

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\alpha_6 &:= 0.0514341663705383 + i\cdot 0.102546382318450.
\end{aligned}$$

- Observe that $\alpha_1\cdot\alpha_2 = \alpha_3\cdot\alpha_4 = \alpha_5\cdot\alpha_6 = 1$
- Computing the even theta constants we notice that for the pairs

$$\left\{\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix},\begin{bmatrix}0 & 1\\0 & 0\end{bmatrix}\right\},\quad \left\{\begin{bmatrix}0 & 0\\1 & 0\end{bmatrix},\begin{bmatrix}0 & 0\\0 & 1\end{bmatrix}\right\},\quad \left\{\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix},\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}\right\},$$

the theta constants coincide

Genus 2 experiments

Our observations lead us to make the following conjecture

Conjecture (Çelik-Fairchild-M., 2022)

The family of hyperelliptic curves corresponding to the family of the translation surfaces, $J_2(\lambda,\mu)$ of genus 2, in the stratum $\mathscr{H}(1,1)$, is given by the equation:

$$y^2 = (x-a)(x-1/a)(x-b)(x-1/b)(x-c)(x-1/c)$$

for some complex parameters a, b, c.

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for some complex parameters a,b,c.

- This suggests that the hyperelliptic curve corresponding to J_2 has an extra involution $(x,y) \mapsto (1/x,y)$
- Exact recovery of these curves may be possible, similar to the work of Silhol and Rodriguez for the L-shape.

J_3 table

n	Time	Approximation
0	0.01	$\begin{pmatrix} i & 0 & 0 \\ 0 & i & 0 \\ 0 & 0 & 2i \end{pmatrix}$
1	0.01	$\begin{pmatrix} -0.163636364+0.972727273i & 0.001474201+0.000245700i & -0.162162162-0.027027027i \\ 0.001474201+0.000245700i & -0.163636364+0.972727273i & -0.162162162-0.027027027i \\ -0.162162162-0.027027027i & -0.162162162-0.027027027i & -0.324324324+1.945945946i \end{pmatrix}$
2	0.01	$\begin{pmatrix} -0.181890640+0.965429514i & 0.000745530+0.000603155i & -0.181145111-0.033967331i \\ 0.000745530+0.000603155i & -0.181890640+0.965429514i & -0.181145111-0.033967331i \\ -0.181145111-0.033967331i & -0.181145111-0.033967331i & -0.362290222+1.932065338i \end{pmatrix}$
3	0.27	$\begin{pmatrix} -0.183837862+0.964626792i & 0.000683710+0.000619978i & -0.183154152-0.034753230i \\ 0.006837101+0.000619978i & -0.183837862+0.964626792i & -0.183154152-0.034753230i \\ -0.183154152-0.034753230i & -0.183154152-0.034753230i & -0.366308303+1.930493539i \end{pmatrix}$
4	2.65	$\begin{pmatrix} -0.184053419+0.964537638i & 0.000676988+0.000621565i & -0.183376430-0.034840796i \\ 0.000676988+0.000621565i & -0.1840534189+0.964537638i & -0.183376430-0.034840796i \\ -0.183376430-0.034840796i & -0.183376430-0.034840796i & -0.366752861+1.930318407i \end{pmatrix}$
5	38.37	$\begin{pmatrix} -0.184077360+0.964527733i & 0.000676243+0.000621738i & -0.183401117-0.034850529i \\ 0.000676243+0.000621738i & -0.184077360+0.964527733i & -0.183401117-0.034850529i \\ -0.183401117-0.034850529i & -0.183401117-0.034850529i & -0.366802234+1.930298942i] \end{pmatrix}$
6	398.04	$\begin{pmatrix} -0.1840800120+0.964526632i & 0.000676160+0.000621757i & -0.183403860-0.034851611i \\ 0.000676160+0.000621757i & -0.1840800120+0.964526632i & -0.183403860-0.034851611i \\ -0.183403860-0.034851611i & -0.183403860-0.034851611i & -0.366807719+1.930296779i \end{pmatrix}$
7	3429.78	$\begin{pmatrix} -0.184080315+0.96452651i & 0.000676151+0.000621759i & -0.183404164-0.034851731i \\ 0.000676151+0.000621759i & -0.184080315+0.96452651i & -0.183404164-0.034851731i \\ -0.183404164-0.034851731i & -0.183404164-0.034851731i & -0.366808329+1.930296539i \end{pmatrix}$

n	Time	Approximation
0	0.01	$\begin{pmatrix} i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & 0 & 0 & 3i \end{pmatrix}$
1	0.01	$\begin{pmatrix} -0.163639+0.972727i & 0.000739+0.000123i & 0.00073859+0.000123i & -0.162162-0.027027i \\ 0.000739+0.000123i & -0.163639+0.972727i & 0.000739+0.000123i & -0.162162-0.027027i \\ 0.000739+0.000123i & 0.000739+0.000123i & -0.163639+0.972727i & -0.162162-0.027027i \\ -0.162162-0.027027i & -0.162162-0.027027i & -0.162162-0.027027i & -0.486486+2.918919i \end{pmatrix}$
2	0.02	$\begin{pmatrix} -0.181893+0.96543i & 0.000374+0.000301i & 0.000374+0.000301i & -0.181145-0.033967i \\ 0.000374+0.000301i & -0.181893+0.965430i & 0.000374+0.000301i & -0.181145-0.033967i \\ 0.000374+0.000301i & 0.000374+0.000301i & -0.181893+0.96543i & -0.181145-0.033967i \\ -0.181145-0.033967i & -0.181145-0.033967i & -0.181145-0.033967i & -0.543435+2.898098i \end{pmatrix}$
3	.48	$\begin{pmatrix} -0.18384+0.964628i & 0.000343+0.00031i & 0.000343+0.00031i & -0.183154-0.034753i \\ 0.000343+0.00031i & -0.18384+0.964628i & 0.000343+0.00031i & -0.183154-0.034753i \\ 0.000343+0.00031i & 0.000343+0.00031i & -0.18384+0.964628i & -0.183154-0.034753i \\ -0.183154-0.034753i & -0.183154-0.034753i & -0.183154-0.034753i & -0.549462+2.89574i \end{pmatrix}$
4	5.70	$\begin{pmatrix} -0.18384+0.964628i & 0.000343+0.00031i & 0.000343+0.00031i & -0.183154-0.034753i \\ 0.0003437+0.00031i & -0.18384+0.964628i & 0.000343+0.00031i & -0.183154-0.034753i \\ 0.000343+0.00031i & 0.000343+0.00031i & -0.18384+0.964628i & -0.183154-0.034753i \\ -0.183154-0.034753i & -0.183154-0.034753i & -0.183154-0.034753i & -0.549462+2.89574i \end{pmatrix}$
5	89.00	$\begin{pmatrix} -0.184079+0.964529i & 0.000339+0.000310i & 0.000339+0.000310i & -0.183401-0.034851i \\ 0.000339+0.000310i & -0.184079+0.964529i & 0.000339+0.0003103i & -0.183401-0.034851i \\ 0.000339+0.000310i & 0.000339+0.000310i & -0.184079+0.964529i & -0.183401-0.034851i \\ -0.183401-0.0348518i & -0.183401-0.034851i & -0.183401-0.0348512i & -0.550203+2.895448i \end{pmatrix}$
6	846.84	$\begin{pmatrix} -0.184082+0.9645278i & 0.0003396+0.000310i & 0.000339+0.000310i & -0.183404-0.034852i \\ 0.000339+0.000310i & -0.184082+0.964527i & 0.000339+0.000310i & -0.183404-0.034852i \\ 0.000339+0.000310i & 0.000339+0.000310i & -0.184082+0.964527i & -0.183404-0.034852i \\ -0.183404-0.034852i & -0.1834044-0.034852i & -0.183404-0.0348512i & -0.550212+2.895445i \end{pmatrix}$
7	7997.04	$\begin{pmatrix} -0.184082+0.964527i & 0.000339+0.000310i & 0.000339+0.000310i & -0.1834043-0.03485125i \\ 0.000339+0.000310i & -0.184082+0.964527i & 0.000339+0.0003105i & -0.1834041-0.0348512i \\ 0.000339+0.000310i & 0.000339+0.000310i & -0.184082+0.964527i & -0.183404-0.034852i \\ -0.183404-0.034852i & -0.183404-0.034852i & -0.183404-0.034852i & -0.550212+2.895445i \end{pmatrix}$

n	Time	Approximation
0	0.01	$\begin{pmatrix} i & 0 & 0 & 0 & 0 \\ 0 & i & 0 & 0 & 0 \\ 0 & 0 & i & 0 & 0 \\ 0 & 0 & 0 & i & 0 \\ 0 & 0 & 0 & 0 & 4i \end{pmatrix}$
1	0.01	$\begin{pmatrix} -0.163639+0.972727i & 0.000737+0.000123i & 0.000003 & 0.000737+0.000123i & -0.162162-0.027027i \\ 0.000737+0.000123i & -0.163639+0.972727i & 0.000737+0.000123i & 0.000003 & -0.162162-0.027027i \\ 0.000003 & 0.000737+0.000123i & -0.163639+0.972727i & 0.000737+0.000123i & -0.162162-0.027027i \\ 0.000737+0.000123i & 0.000003 & 0.000737+0.000123i & -0.163639+0.972727i & -0.162162-0.027027i \\ -0.162162-0.027027i & -0.162162-0.027027i & -0.162162-0.027027i & -0.162162-0.027027i & -0.648649+3.891892i \end{pmatrix}$
2	0.03	$\begin{pmatrix} -0.181893+0.96543i & 0.000373+0.000302i & 0.000002-0.000001i & 0.0003731+0.000303i & -0.181145-0.033967i \\ 0.000373+0.000302i & -0.181893+0.96543i & 0.000373+0.000302i & 0.000002-0.000001i & -0.181145-0.033967i \\ 0.000002-0.000001i & 0.000373+0.000302i & -0.181893+0.965430i & 0.000373+0.000302i & -0.181145-0.033967i \\ 0.000373+0.000302i & 0.000002-0.000001i & 0.000373+0.000302i & -0.1818938+0.96543i & -0.181145-0.033967i \\ -0.181145-0.033967i & -0.181145-0.033967i & -0.181145-0.033967i & -0.181145-0.033967i & -0.72458+3.864131i \end{pmatrix}$
3	0.69	$\begin{pmatrix} -0.18384+0.964628i & 0.000342+0.00031i & 0.000002-0.0000001i & 0.000342+0.00031i & -0.183154-0.034753i \\ 0.000342+0.00031i & -0.18384+0.964628i & 0.000342+0.00031i & 0.000002-0.000001i & -0.183154-0.034753i \\ 0.000002-0.000001i & 0.000342+0.00031i & -0.18384+0.964628i & 0.000342+0.00031i & -0.183154-0.034753i \\ 0.000342+0.00031i & 0.000002-0.000001i & 0.000342+0.00031i & -0.18384+0.964628i & -0.183154-0.034753i \\ -0.183154-0.034753i & -0.183154-0.034753i & -0.183154-0.034753i & -0.183154-0.034753i & -0.732617+3.860987i \end{pmatrix}$
4	9.02	$\begin{pmatrix} -0.184055+0.964538i & 0.000338+0.000311i & 0.000002-0.000001i & 0.000338+0.000311i & -0.183376-0.034841i \\ 0.000338+0.000311i & -0.184055+0.964538i & 0.000338+0.000311i & 0.000002-0.000001i & -0.183376-0.034841i \\ 0.000001-0.000001i & 0.000338+0.000311i & -0.184055+0.964538i & 0.000338+0.000311i & -0.183376-0.034841i \\ 0.000338+0.000311i & 0.000001-0.000001i & 0.000338+0.000311i & -0.184055+0.964538i & -0.183376-0.034841i \\ -0.183376-0.034841i & -0.183376-0.034841i & -0.183376-0.034841i & -0.183376-0.034841i & -0.733506+3.860637i \end{pmatrix}$
5	94.25	$\begin{pmatrix} -0.184079+0.964529i & 0.000338+0.000311i & 0.000001-0.000001i & 0.000338+0.000311i & -0.183401-0.034851i \\ 0.000338+0.000311i & -0.184079+0.964529i & 0.000338+0.000311i & 0.000002-0.000001i & -0.183401-0.034851i \\ 0.000002-0.000001i & 0.000338+0.000311i & -0.184079+0.964529i & 0.000338+0.000311i & -0.183401-0.034851i \\ 0.000338+0.000311i & 0.000002-0.000001i & 0.000338+0.000311i & -0.184079+0.964529i & -0.183401-0.034851i \\ -0.183401-0.034851i & -0.183401-0.034851i & -0.183401-0.034851i & -0.183401-0.034851i & -0.733604+3.860598i \end{pmatrix}$
6	1051.54	$\begin{pmatrix} -0.184082+0.964527i & 0.000338+0.000311i & 0.000002-0.000001i & 0.000338+0.000311i & -0.183404-0.034852i \\ 0.000338+0.000311i & -0.184082+0.964527i & 0.000338+0.000311i & 0.000002-0.000001i & -0.183404-0.034852i \\ 0.000002-0.000001i & 0.000338+0.000311i & -0.184082+0.964527i & 0.000338+0.000311i & -0.183404-0.034852i \\ 0.000338+0.000311i & 0.000002-0.000001i & 0.000338+0.000311i & -0.184082+0.964527i & -0.183404-0.034852i \\ -0.183404-0.034852i & -0.183404-0.034852i & -0.183404-0.034852i & -0.183404-0.034852i & -0.733615+3.860594i \end{pmatrix}$
7	12739.99	$\begin{pmatrix} -0.184082+0.964527i & 0.000338+0.000311i & 0.000002-0.000001i & 0.000338+0.000311i & -0.183404-0.034852i \\ 0.000338+0.000312i & -0.184082+0.964527i & 0.000338+0.000311i & 0.000002-0.000001i & -0.183404-0.034852i \\ 0.000002-0.000001i & 0.000338+0.000311i & -0.184082+0.964527i & 0.000338+0.000311i & -0.183404-0.034852i \\ 0.000338+0.000311i & 0.000002--0.000001i & 0.000338+0.000311i & -0.184082+0.964527i & -0.183404-0.034852i \\ -0.183404-0.034852i & -0.183404-0.034852i & -0.183404-0.034852i & -0.183404-0.034852i & -0.733617+3.860593i \end{pmatrix}$

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- Are the approximating Riemann matrices (for $g=4,5$ in the Schottky locus?)
  - When $g=4$ yes, when $g=5$ they are within the weak Schottky locus.

Thank you!