

# Qualifying Exam Syllabus

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April 17th, 3:00, on Zoom

Committee: Sylvie Corteel (Exam Chair), Bernd Sturmfels (Advisor), Olga Holtz, Jim Pitman.

## 1 Major topic: Algebraic Combinatorics (Algebra)

References: Art of Counting, Sagan. Chapters 1-4, 7, 8

- **Enumeration** Permutations, trees, partitions, lattice paths, pattern avoidance, inclusion-exclusion, sign reversing involutions, reflection principle, Lindström-Gessel-Viennot Lemma, Matrix Tree Theorem
- **Generating Functions** ordinary generating functions,  $q$ -analogs, recurrence relations, exponential generating functions, labeled structures, exponential formula, Lagrange Inversion,
- **Symmetric Functions** Young Tableaux, Sym, Schur basis,  $P$ -partitions, hooklengths, RSK correspondence and algorithm, generalizations of Schur polynomials
- **Quasisymmetric Functions** QSym, reverse  $P$ -partitions, Mobius Inversion.

## 2 Nonlinear Algebra (Algebra)

References: Invitation to Nonlinear Algebra, Michalek and Sturmfels. Chapters 1-12

- **Polynomials:** Ideals, Gröbner Bases, Hilbert function, Hilbert Series, Hilbert Polynomial, dimension, degree
- **Varieties:** Affine varieties, Zariski topology, closure, Bézout's theorem, projective varieties, genus
- **Solving and Decomposing:** zero-dimensional ideals, primary decomposition, linear PDE with constant coefficients
- **Mapping and Projecting:** elimination, matrix completion, Sylvester matrix, resultants, image of a polynomial map
- **Linear Spaces and Grassmannians** Grassmannian, Plücker relations, Schubert calculus, degree of Grassmannian
- **Nullstellensätze** certificates for infeasibility, Extended Buchberger Algorithm, Hilbert's Nullstellensatz, real Nullstellensatz, Artin's Theorem
- **Tropical Algebra** arithmetic and valuations, linear algebra over the tropical semiring, tropical varieties

- **Toric Varieties** algebraic torus, character, affine toric varieties, projective toric varieties, polytopes, choosing coordinates
- **Tensors** spectral decomposition, singular value decomposition, tensor decomposition, eigenvalues, tensor rank, hyperdeterminant, matrix multiplication
- **Representation Theory** representations, irreducible representations, Maschke's Theorem, Schur's Lemma, character, Schur polynomials, Schur-Weyl duality, using symmetry
- **Invariant Theory** Hilbert's Finiteness Theorem, Reynolds operator, geometric invariant theory, Derksen's Algorithm
- **Semidefinite Programming** spectrahedra, SDP, sums of squares

### 3 Minor topic: Probability Theory (Probability)

References: Probability: Theory and Examples, Durrett chapters 1-3

- **Basic Notions:** Measure Theory,  $\pi - \lambda$  Theorem, Random Variables, Theory of Integration, Expected Value and Variance, Inequalities, Change of Variables, Fubini's Theorem, Notions of Convergence of Random Variables
- **Laws of Large Numbers:** Independence, Convolutions, Weak Laws of Large Numbers, Borel-Cantelli Lemmas, Strong Law of Large Numbers, Convergence of Random Series, Kolmogorov 0-1 Law
- **Central Limit Theorems:** Properties of Weak Convergence, Helly's Selection Theorem, Characteristic Functions, CLT for i.i.d. Sequences, CLT for Triangular Arrays