Qualifying Exam Syllabus

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April 17th, 3:00, on Zoom

Committee: Sylvie Corteel (Exam Chair), Bernd Sturmfels (Advisor), Olga Holtz, Jim Pitman.

1 Major topic: Algebraic Combinatorics (Algebra)

References: Art of Counting, Sagan. Chapters 1-4, 7, 8

- **Enumeration** Permutations, trees, partitions, lattice paths, pattern avoidance, inclusion-exclusion, sign reversing involutions, reflection principle, Lindström-Gessel-Viennot Lemma, Matrix Tree Theorem

- **Generating Functions** ordinary generating functions, \(q\)-analogs, recurrence relations, exponential generating functions, labeled structures, exponential formula, Lagrange Inversion,

- **Symmetric Functions** Young Tableaux, Sym, Schur basis, \(P\)-partitions, hooklengths, RSK correspondence and algorithm, generalizations of Schur polynomials

- **Quasisymmetric Functions** QSym, reverse \(P\)-partitions, Mobius Inversion.

2 Nonlinear Algebra (Algebra)

References: Invitation to Nonlinear Algebra, Michalek and Sturmfels. Chapters 1-12

- **Polynomials**: Ideals, Gröbner Bases, Hilbert function, Hilbert Series, Hilbert Polynomial, dimension, degree

- **Varieties**: Affine varieties, Zariski topology, closure, Bézout’s theorem, projective varieties, genus

- **Solving and Decomposing**: zero-dimensional ideals, primary decomposition, linear PDE with constant coefficients

- **Mapping and Projecting**: elimination, matrix completion, Sylvester matrix, resultants, image of a polynomial map

- **Linear Spaces and Grassmannians** Grassmannian, Plücker relations, Schubert calculus, degree of Grassmannian

- **Nullstellensätze** certificates for infeasibility, Extended Buchberger Algorithm, Hilbert’s Nullstellensatz, real Nullstellensatz, Artin’s Theorem

- **Tropical Algebra** arithmetic and valuations, linear algebra over the tropical semiring, tropical varieties
• Toric Varieties algebraic torus, character, affine toric varieties, projective toric varieties, polytopes, choosing coordinates

• Tensors spectral decomposition, singular value decomposition, tensor decomposition, eigenvalues, tensor rank, hyperdeterminant, matrix multiplication

• Representation Theory representations, irreducible representations, Maschke’s Theorem, Schur’s Lemma, character, Schur polynomials, Schur-Weyl duality, using symmetry

• Invariant Theory Hilbert’s Finiteness Theorem, Reynolds operator, geometric invariant theory, Derksen’s Algorithm

• Semidefinite Programming spectrahedra, SDP, sums of squares

3 Minor topic: Probability Theory (Probability)

References: Probability: Theory and Examples, Durrett chapters 1-3

• Basic Notions: Measure Theory, \( \pi - \lambda \) Theorem, Random Variables, Theory of Integration, Expected Value and Variance, Inequalities, Change of Variables, Fubini’s Theorem, Notions of Convergence of Random Variables

• Laws of Large Numbers: Independence, Convolutions, Weak Laws of Large Numbers, Borel-Cantelli Lemmas, Strong Law of Large Numbers, Convergence of Random Series, Kolmogorov 0-1 Law

• Central Limit Theorems: Properties of Weak Convergence, Helly’s Selection Theorem, Characteristic Functions, CLT for i.i.d. Sequences, CLT for Triangular Arrays