Week 9: Midterm review

- definition (different from a set)
- bounded, \text{inf}/\text{sup}
- limits:
  - existence definition \( \varepsilon-N \)
  - ways to show limits
    - algebraic operations
    - Monotone sequences
  - Cauchy criterion
  - ways to show non-existence of limits
    - negation of definition \( \neg \)
- Subsequences:
  - definition
    - Bolzano-Weierstrass
    - relation to lim sup, lim inf
    - set of all possible subsequence limits
    - relation to convergence
- Series:
  - partial sum is a sequence
  - convergence tests:
    - Ratio/root/Integral test
    - Alternating sum
  - absolute vs. conditional convergence
- Domain
  - Continuity
    - Definition: Sequential vs. "ε-δ"
    - Negation
  - How to show continuity or discontinuity
  - Arithmetic operations and composition of continuous functions
  - Properties: Max/min on [a, b]
    - Intermediate value property
    - Monotone continuous function on an interval
- Uniform continuity
  - Definition and negation
  - Continuous functions on [a, b]
  - Extension from (a, b) to [a, b]
  - Relation to derivatives
- Convergence of functions \( \mathbb{N} \times \mathbb{R} \rightarrow \mathbb{R} 
  - Pointwise: how to find the limit
  - Uniform convergence: definition and negation
  - Exchange of continuity/sup/derivative/integral with taking limits
Some other topics
  - Induction
  - Sets operations
  - Completeness of $\mathbb{R}$, Archimedean property
  - Proof format