Worksheet 8 (Feb. 8)

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1 Review

DEFINITIONS

- composition of linear transformation, matrix product;

- Inverse of linear transformation, inverse of matrix;

Remark 1. Let A be the matrix of S and B be of T. The matrix of the composition $T \circ S$ is then just BA. In this sense, not any two matrices A and B can be composed, as not any two linear transformations can be composed. They only can when the number of columns of the former is equal to the number of rows of the latter.

2 Problems

Example 1. Determine the injectivity and surjectivity of the composition $T \circ S$ (note that this means to first apply S and then apply T!!!) of linear transformations T and S, where
(1) $S : \mathbb{R}^2 \to \mathbb{R}^2$ is the rotation counterclockwise by $\pi/4$, and $T : \mathbb{R}^2 \to \mathbb{R}^2$ is the projection to y-axis;

(2) $S : \mathbb{R} \to \mathbb{R}^2$ satisfies $S(1) = (2, 0)$ and $T : \mathbb{R}^2 \to \mathbb{R}$ is the projection to x-axis;

(3) the matrices of S and T are

$$S = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \end{pmatrix}, T = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}.$$

Example 2. Find counterexamples to the following statements.
(1) The composition $T \circ S$ of two linear transformations is injective, then both T and S are injective.

(2) The composition $T \circ S$ of two linear transformations is injective, then both T and S are injective.

(3) If $n \times n$ matrices satisfy $AB = AC$, then $B = C$.

Example 3. Compute the matrix product

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} \cdot \begin{pmatrix} 6 & 5 \\ 4 & 3 \\ 2 & 1 \end{pmatrix}$$

Example 4. *This one looks like a proof problem but it is actually about computation.* Let A and B be two 2×2 matrices and let

$$C = AB - BA.$$

Show that the sum of the two diagonal entries of C, i.e. $C_{11} + C_{22}$, is zero.