Worksheet 7 (Feb. 5)

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1 Review

DEFINITIONS

- *Recall:* matrix of a linear transformation;
- range and kernel of a linear transformation;
- one-to-one (injective) linear transformation, onto (surjective) linear transformation, bijective linear transformation.

METHODS AND IDEAS

[For the complete version see P7 of the professor's notes of Lecture 6. Let A be an $m \times n$ matrix.]

- Further expanded **criterion** for $\geqslant 1$ solution (existence): The linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m, T(\mathbf{x}) = A\mathbf{x}$ is surjective $\Leftrightarrow$ the linear system $A\mathbf{x} = \mathbf{b}$ is consistent for any $\mathbf{b} \in \mathbb{R}^m$.
- Further expanded **criterion** for $\leqslant 1$ solution (inconsistency or uniqueness): The linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m, T(\mathbf{x}) = A\mathbf{x}$ is injective $\Leftrightarrow$ the linear system $A\mathbf{x} = \mathbf{b}$ has $\leqslant 1$ solution for any $\mathbf{b} \in \mathbb{R}^m$.

2 Problems

Example 1. Find a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ that is
(a) one-to-one and onto;
(b) one-to-one but not onto;
(c) onto but not one-to-one;
(d) neither one-to-one nor onto.

Example 2. True or false.

( ) A linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$ is completely determined by its effect on the columns of the $n \times n$ identity matrix.

( ) A linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$ is one-to-one if it maps each $\mathbf{x} \in \mathbb{R}^n$ to a unique vector in $\mathbb{R}^m$.

( ) If a linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$ satisfies $T(\mathbf{e}_1 + \mathbf{e}_n) = \mathbf{0}$, then T is not injective.

( ) A linear transformation is onto if and only if its matrix has a pivot in each row.

( ) A linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$ is surjective if $n > m$.

( ) A linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$ is surjective only if $n > m$.

Example 3. *'Find values of c' cliché.* Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation associated to the matrix

$$A = \begin{pmatrix} 3 & 1 & 3 \\ c & 2 & 6 \\ 1 & 0 & -1 \end{pmatrix}.$$

(1) When is T injective? (2) When is T surjective?