Worksheet 5 (Feb. 1)

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1 Review

Recall from last time

- linear independence, how to check;

- matrix-vector product.

Theorem 1. (Solution of inhomogeneous linear system)
Let $\mathbf{x} = \mathbf{x}_{prt}$ *be one particular solution of* $A\mathbf{x} = \mathbf{b}$*, then any solution of* $A\mathbf{x} = \mathbf{b}$ *can be written as*

$$\mathbf{x} = \mathbf{x}_{prt} + \mathbf{x}_{hmg},$$

for some solution $\mathbf{x}_{hmg}$ *of the homogeneous system* $A\mathbf{x} = \mathbf{0}$*, and vice versa.*

2 Problems

Example 1. True or false.

( ) The columns of any 4×5 matrix are linearly dependent.

( ) The columns of a matrix A are linearly dependent if the equation $A\mathbf{x} = \mathbf{0}$ is consistent.

( ) If A is a 2×5 matrix with two pivot positions, $A\mathbf{x} = \mathbf{b}$ is consistent for any $b \in \mathbb{R}^2$.

( ) Two vectors are linearly dependent if and only if geometrically they lie on the same line through the origin.

( ) The vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$ are linearly dependent if $\mathbf{v}_3 = \mathbf{0}$.

( ) If the vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ are linearly dependent, then $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$ are linearly dependent for any $\mathbf{v}_4$.

Example 2. Consider

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 1 \\ 2 \end{bmatrix},$$

and solve the two linear systems

$$(a) A\mathbf{x} = \mathbf{0}. \qquad\qquad (b) A\mathbf{x} = \mathbf{b}.$$

Example 3. Compute the matrix-vector multiplication

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} \begin{pmatrix} 7 \\ 8 \\ 9 \end{pmatrix}.$$

Remark 1. *With a view toward future lectures:* Regarding vector as a $n \times 1$ matrix, then we actually defined the product of an $m \times n$ matrix and an $n \times 1$ matrix as an $m \times 1$ matrix. (In our example above, $m = 2$ and $n = 3$.) Can we generalize the definition to multiplication of an $m \times n$ matrix with an $n \times p$ matrix? The answer should be $m \times p$.