

Worksheet 32 (April 30)

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1 Review

METHODS AND IDEAS

Theorem 1. (Convergence theorem)

Let $f(x)$ be a piece-wise continuous function on $[-\pi, \pi]$ and $FS_f(x)$ be its Fourier expansion, then $FS_f(x) = f(x^+) + f(x^-)$.

Remark 1. In particular, if f is continuous at x , which means $f(x^+) = f(x^-)$, $FS_f(x) = f(x)$. This recovers our claim last time that FS_f is equal to f almost everywhere.

Corollary 1. (Functions defined on \mathbb{R})

Let $g(x)$ be a 2π -periodic function on \mathbb{R} such that both $g(x)$ and $g'(x)$ are continuous, then $FS_g(x) = g(x)$.

Corollary 2. (Fourier expansion of the derivative) Let f be a function on $[-\pi, \pi]$ such that $f(\pi^-) = f(-\pi^+)$ and f, f' and f'' are all piece-wise continuous. If

$$FS_f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + b_n \sin nx,$$

then

$$FS_{f'}(x) = \sum_{n=1}^{\infty} -na_n \sin nx + nb_n \cos nx.$$

Note that we need the second differentiability of f as part of our conditions.

2 Problems

Example 1. Find $FS_f(x)$ without computing the Fourier coefficients, for

$$f(x) = \max\{x, 1\} \quad x \in [-\pi, \pi].$$

Example 2. Find the Fourier coefficients of the 2π -periodic function

$$f(x) = |\sin x| \quad x \in \mathbb{R}.$$

Example 3. Find the expansion of

$$f(x) = x$$

in terms of $\{\sin n\pi x\}_{n \in \mathbb{N}^+}$.