

Worksheet 25 (April 12)

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1 Preview

METHODS AND IDEAS

Theorem 1. Let A be an $m \times n$ matrix, $\sigma_1 \geq \sigma_2 \geq \dots \geq \sigma_r > 0$ be the singular values of A , and

$$A = \sigma_1 \mathbf{u}_1 \mathbf{v}_1^T + \dots + \sigma_r \mathbf{u}_r \mathbf{v}_r^T$$

be the singular value decomposition of A . Then $\text{rank } A = r$, and for any $i = 1, 2, \dots, r$,

$$A_i = \sigma_1 \mathbf{u}_1 \mathbf{v}_1^T + \dots + \sigma_i \mathbf{u}_i \mathbf{v}_i^T$$

is the best approximation of A by rank = i matrices.

2 Problems

Example 1. True or false.

- () The singular values of A^T are the same as the singular values of A for any A .
- () If a square matrix A is symmetric, so is A^2 .
- () If a square matrix A is symmetric, so is A^{-1} .
- () If $(1, 2)^T$ and $(2, 1)^T$ are both eigenvectors of a symmetric 2×2 matrix A , so is $(2, 2)^T$.

3 Some Calculus

DERIVATIVES

$f(x)$	$f'(x)$
e^x	
$a_n x^n + a_{n-1} x^{n-1} + \dots + a_0$	
$\ln x$	
$\sin x, \cos x$	

Useful properties of derivatives:

- $(f \pm g)' = f' \pm g'$;
- $(f \cdot g)' = f' \cdot g + f \cdot g'$;
- $(f/g)' = (f' \cdot g - f \cdot g')/g^2$;
- Denote by h the composition of f and g , i.e. $h(x) = f(g(x))$, then $h' = f'(g) \cdot g'$.

INTEGRALS

$f(x)$	$\int f(x)dx$	$\int_0^x f(s)ds$
e^x		
x^n ($n \neq -1$)		
x^{-1}		
$\sin x, \cos x$		

EXERCISES

Example 2. Find the derivatives of the functions below:

$$x^3 e^2, \tan x, \ln x^2, x \sin \frac{1}{x}.$$

Example 3. Find all functions $f = f(x)$ such that $f(x) = f'(x)$.