Worksheet 21 (March 29)

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1 Problems

Example 1. True or false.

( ) Let $W \subset \mathbb{R}^n$ be a subspace and $\mathbf{a}_1, \ldots, \mathbf{a}_k$ spans W. If $\mathbf{x}$ is orthogonal to each $\mathbf{a}_i (i = 1, 2, \ldots, k)$, then $\mathbf{x} \in W^\perp$.

( ) Let $W \subset \mathbb{R}^n$ be a subspace, $\mathbf{w} \in W$ and $\mathbf{x} \in \mathbb{R}^n$, then $(\mathrm{Proj}_W \mathbf{x}) \cdot \mathbf{w} = \mathbf{x} \cdot \mathbf{w}$.

( ) The orthogonal complement of $\mathrm{Col}(A)$ is the solution set of $A\mathbf{x} = \mathbf{b}$.

Example 2. Let $\mathbf{u}, \mathbf{v}, \mathbf{w}$ be three unit vectors in $\mathbb{R}^n$ satisfying $\mathbf{u} + \mathbf{v} + \mathbf{w} = \mathbf{0}$. Prove that $\mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{w} = \mathbf{w} \cdot \mathbf{u} = -1/2$.

Example 3. Find the orthogonal complement of the subspace $W \subset \mathbb{R}^3$ spanned by

$$\mathbf{w}_1 = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}, \mathbf{w}_2 = \begin{pmatrix} -1 \\ -1 \\ -2 \end{pmatrix}.$$

Example 4. Let $\mathbf{u}$ and $\mathbf{v}$ be two vectors in $\mathbb{R}^3$ whose orthongonal projection to the subspace W are $(1, 1, -1)^T$ and $(2, -4, 1)^T$ respectively.
(a) What is the orthogonal projection of $\mathbf{u} + 2\mathbf{v}$ to W?
(b) What is the smallest possible value of $\|\mathbf{u} - \mathbf{v}\|$?
(c) What is W?