Worksheet 2 (Jan. 25)

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1 Review

DEFINITIONS

- matrix, augmented matrix ($\_\_ \times \_\_$), coefficient matrix ($\_\_ \times \_\_$);

- three types of row reductions: scaling, interchange, replacement;

- REF, RREF, pivot column, pivot position;

- existence and uniqueness of solutions.

METHODS AND IDEAS

- To solve linear systems, we transform them into simpler systems. This can be done by writing the coefficients and constants of a system into a matrix (called the **augmented matrix**), and then doing row reductions to transform the matrix into its **row Echelon form**, or even its **row reduced Echelon form**. [For the algorithm see P12 of the professor's lecture notes 2.]
- *(Criterion of existence and uniqueness)* Solutions of a linear system **exist** if and only if the last column (the constant column) of REF of the augmented matrix is not pivot. The solution **uniquely exists** if and only if there are no free variables, i.e. all except the last column of REF are pivot.
- Note that we need only **REF** but not RREF to determine existence and uniqueness.

2 Problems

Example 1. *We will do this together.* Solve the linear systems below, by writing out the augmented matrix and applying appropriate row reductions.

$$\text{(a)} \begin{cases} x_1 + \ \ x_2 + 3x_3 = 3 \\ 2x_1 + 2x_2 - \ \ x_3 = -1 \\ x_1 + 3x_2 + 5x_3 = -5 \end{cases}, \qquad \text{(b)} \begin{cases} x_1 + \ \ x_2 + 3x_3 = 3 \\ 2x_1 + 2x_2 - \ \ x_3 = -1 \end{cases}.$$

Example 2. True or false.

( ) A linear system is inconsistent if it has at least two different solutions.

( ) A linear system with more than 1 solution has infinitely many solutions.

( ) Whenever a linear system has free variables, the solution set contains at least two solutions.

( ) Every elementary row operation is reversible.

( ) Two matrices are row equivalent if they have the same number of rows.

( ) The row echelon form (REF) of a matrix is unique.

Example 3. What do we know of the consistency of a linear system
(a) whose augmented matrix is 3×5 with the 5th column being pivot?
(b) whose coefficient matrix is 3×5 with 2 pivot columns?
(c) whose coefficient matrix is 3×5 with 3 pivot columns?

Example 4. Find all intersection points of the following three planes in $\mathbb{R}^3$

$$x_1 + x_2 + 3x_3 = 3, 2x_1 + 2x_2 - x_3 = -1, x_1 + 3x_2 + 5x_3 = -5.$$

Hint: Recall Example 1.

Remark 1. What else can the intersection of three planes in $\mathbb{R}^3$ look like? If this is too hard, what can the intersection of two lines in $\mathbb{R}^2$ look like?