Worksheet 19 (March 17)

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1 Review

DEFINITIONS

- eigenvalue and eigenvector of linear transformation;
- complex number, conjugate, absolute value;
- complex eigenvalue, complex eigenvector.

2 Problems

Example 1. Find the complex eigenvalues of the matrix

$$A = \begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}.$$

Then find a basis of $\mathbb{R}^2$ under which the transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a scaling followed by rotation.

Example 2. Find an example or disprove existence:
(a) a real 3×3 matrix A whose only real eigenvalue is 1 with algebraic multiplicity 2.

(b) a real 3×3 invertible matrix B with 2 and 3 being two of its eigenvalues, and 4 being an eigenvalue of its inverse.

(c) a non-diagonal 2×2 matrix C with eigenvalues 2 and 4, and determinant 6.

(d) a 3×3 matrix F with eigenvalues 2 and 3 such that F^2 is not diagonalizable.

Example 3. Consider the linear transformation $T : \mathbb{P}_2 \to \mathbb{P}_2$ given by $T(f(x)) = f'(x) + f(x)$. Find all eigenvalues and eigenvectors of T.