

Worksheet 18 (March 15)

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1 Problems

Example 1. True or false.

- () If A and B are similar, $\chi_A(\lambda) = \chi_B(\lambda)$.
- () If A and B are similar, then an eigenvector of A also an eigenvector of B .
- () If A is a 6×6 matrix which has 3 distinct eigenvalues with geometric multiplicities 3, 2 and 1, then A is diagonalizable.
- () If square matrices A, P and D satisfies $AP = PD$ and D is diagonal, then nonzero columns of P are eigenvectors of A .

Example 2. Compute

$$\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}^n.$$

The result may be used to compute the general formula of the Fibonacci sequence F_n , defined by $F_n = F_{n-1} + F_{n-2}$ and $F_1 = F_2 = 1$, but we will not discuss this application here.

Example 3. Let A and B be two $n \times n$ matrices such that $AB = BA$ and A has n distinct eigenvalues. Let $\mathbf{v} \in \mathbb{R}^n$ be an eigenvector of A . Prove that \mathbf{v} is also an eigenvector of B .