

Worksheet 8 (Feb. 8)

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1 Review

DEFINITIONS

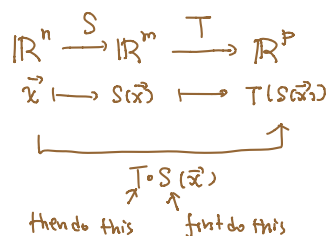
- composition of linear transformation, matrix product;

Computation. $B_{p \times m} A_{m \times n} = (B \vec{a}_1 \ B \vec{a}_2 \ \dots \ B \vec{a}_n)_{p \times n}$
 Where $A = (\vec{a}_1 \ \dots \ \vec{a}_n)$.

- Inverse of linear transformation, inverse of matrix;

Rmk. $B \cdot A$ is only defined for B and A satisfying

columns of B = # rows of A .



$$\begin{aligned} T \circ S(\vec{x}) &= T(S(\vec{x})) = T(A \cdot \vec{x}) \\ &= B \cdot (A \cdot \vec{x}) \\ &= (B \cdot A) \cdot \vec{x}. \end{aligned}$$

Remark 1. Let A be the matrix of S and B be of T . The matrix of the composition $T \circ S$ is then just BA . In this sense, not any two matrices A and B can be composed, as not any two linear transformations can be composed. They only can when the number of columns of the former is equal to the number of rows of the latter.

$$\begin{aligned} S(\vec{x}) &= A_{m \times n} \vec{x} \\ T(\vec{y}) &= B_{p \times m} \vec{y} \end{aligned}$$

2 Problems

Example 1. Determine the injectivity and surjectivity of the composition $T \circ S$ (note that this means to first apply S and then apply T !!!) of linear transformations T and S , where

(1) $S : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is the rotation counterclockwise by $\pi/4$, and $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is the projection to y -axis;

(2) $S : \mathbb{R} \rightarrow \mathbb{R}^2$ satisfies $S(1) = (2, 0)$ and $T : \mathbb{R}^2 \rightarrow \mathbb{R}$ is the projection to x -axis;

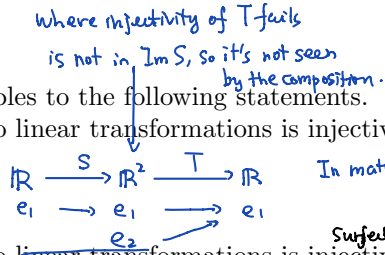
(3) the matrices of S and T are

$$S = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \end{pmatrix}, T = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}.$$

Example 2.

Find counterexamples to the following statements.

(1) The composition $T \circ S$ of two linear transformations is injective, then both T and S are injective.



In matrix, $S = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $T \circ S = (1)$
 $T = (1 \ 1)$ "identity map"

Correct version If $T \circ S$ is injective, then S is injective, but not necessarily T .

(2) The composition $T \circ S$ of two linear transformations is injective, then both T and S are injective.

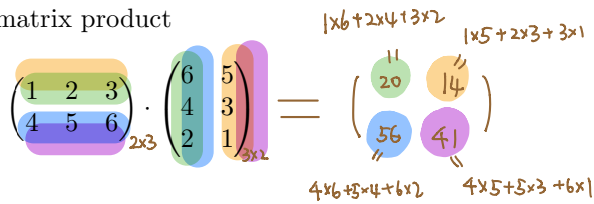
the same example in (1) works:
 S is not surjective, T is surjective.

Correct version If $T \circ S$ is injective.

(3) If $n \times n$ matrices satisfy $AB = AC$, then $B = C$.

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, B = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, C = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

Example 3. Compute the matrix product



Example 4. This one looks like a proof problem but it is actually about computation. Let A and B be two 2×2 matrices and let

$$C = AB - BA.$$

Show that the sum of the two diagonal entries of C , i.e. $C_{11} + C_{22}$, is zero.