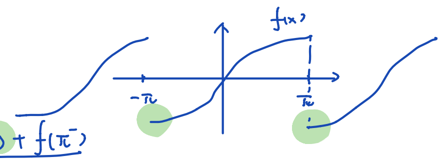
Worksheet 32 (April 30)

DIS 119/120 GSI Xiaohan Yan



$FS_f(\pi) = \frac{f(\pi^+) + f(\pi^-)}{2}$

$\parallel \quad = \frac{f(-\pi^+) + f(\pi^-)}{2}$

$FS_f(-\pi) = \frac{f(-\pi^+) + f(\pi^-)}{2}$

1 Review

METHODS AND IDEAS

Theorem 1. (Convergence theorem)
Let $f(x)$ be a piece-wise continuous function on $[-\pi, \pi]$ and $FS_f(x)$ be its Fourier expansion, then $FS_f(x) = \frac{f(x^+) + f(x^-)}{2}$.

Remark 1. In particular, if f is continuous at x, which means $f(x^+) = f(x^-)$, $FS_f(x) = f(x)$. This recovers our claim last time that FS_f is equal to f almost everywhere.

Corollary 1. (Functions defined on $\mathbb{R}$)
Let $g(x)$ be a 2π-periodic function on $\mathbb{R}$ such that both $g(x)$ and $g'(x)$ are continuous, then $FS_g(x) = g(x)$.

Corollary 2. (Fourier expansion of the derivative) *Let f be a function on $[-\pi, \pi]$ such that $f(\pi^-) = f(-\pi^+)$ and f, f' and f'' are all piece-wise continuous. If*

$$FS_f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + b_n \sin nx,$$

then

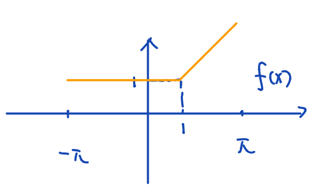
$$FS_{f'}(x) = \sum_{n=1}^{\infty} -na_n \sin nx + nb_n \cos nx.$$

Note that we need the second differentiability of f as part of our conditions.

2 Problems

Example 1. Find $FS_f(x)$ without computing the Fourier coefficients, for

$$f(x) = \max\{x, 1\} \qquad x \in [-\pi, \pi].$$



→ $\forall\, x \in (-\pi, \pi)$ $f(x)$ is continuous at x.

so $FS_f(x) = f(x) = \max\{x, 1\}$

→ $x = -\pi, \pi$. $FS_f(\pi) = FS_f(-\pi) = \frac{1+\pi}{2}$

Example 2. Find the Fourier coefficients of the 2π-periodic function

$$f(x) = |\sin x| \qquad\qquad x \in \mathbb{R}.$$

Example 3. Find the expansion of

$$f(x) = x$$

in terms of $\{\sin n\pi x\}_{n\in\mathbb{N}^+}$.