Worksheet 27 (April 16)

DIS 119/120 GSI Xiaohan Yan

1 Review

DEFINITIONS

- Wronskian; y_1, y_2 two solutions of 2^{nd} order ODE

  $W[y_1,y_2](t) = \det \begin{pmatrix} y_1(t) & y_2(t) \\ y_1'(t) & y_2'(t) \end{pmatrix}$

- systems of first-order linear ODE. $\vec{x}(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{pmatrix}$

  We will consider only the homog. equations. $\vec{f}(t)=0$

  $\vec{x}'(t) = \underset{n\times n \text{ matrix}}{A}\vec{x}(t) + \vec{f}(t)$. $\vec{f}(t) = \begin{pmatrix} f_1(t) \\ \vdots \\ f_n(t) \end{pmatrix}$

THEOREMS

Theorem 1. *For any $n \times n$ matrix A, the (homogeneous) system of first-order ODE*

$$\mathbf{x}'(t) = A\mathbf{x}(t)$$

has an n-dimensional solution set $\mathrm{Ker}(T)$*, and the linear transformation S :* $\mathrm{Ker}(T) \to \mathbb{R}^n$ *defined by $S(\mathbf{x}(t)) = \mathbf{x}(0)$ is an isomorphism. In other words, the initial value problem*

$$\mathbf{x}'(t) = A\mathbf{x}(t), \mathbf{x}(0) = \mathbf{x}_0 \in \mathbb{R}^n$$

has unique solutiion for any given $\mathbf{x}_0$.

2 Problem

Example 1. True or false.

(F) Two functions $y_1(t)$ and $y_2(t)$ are linearly independent if and only if the Wronskian $W[y_1, y_2](t)$ is nonzero for all t. $y_1(t), y_2(t)$ should be solutions of the same 2^{nd} ODE $ay''+by'+cy=0$

(T) If $y_1(t)$ and $y_2(t)$ are two solutions of $ay'' + by' + cy = 0$ and $W[y_1, y_2](0) \neq 0$, then $W[y_1, y_2](t) \neq 0$ for all t.

$\Downarrow$

$\begin{pmatrix} y_1(0) \\ y_1'(0) \end{pmatrix}$ & $\begin{pmatrix} y_2(0) \\ y_2'(0) \end{pmatrix}$ are linearly independent.

$\Downarrow$

$y_1(t), y_2(t)$ are L.I.

$\Downarrow$

$\begin{pmatrix} y_1(t) \\ y_1'(t) \end{pmatrix}$ & $\begin{pmatrix} y_2(t) \\ y_2'(t) \end{pmatrix}$ the initial values at any pt t are L.I.

$\Downarrow$

$W[y_1,y_2](t) \neq 0$, at any pt t

Skipped ( ) If $ay'' + by' + cy = 0$ has a bounded solution, then $b = 0$.

$$\begin{cases} x_1'(t) = x_1(t) + x_2(t) + \sin t \\ x_2'(t) = 2x_1(t) - 2x_2(t) + e^t \end{cases}$$

$x'(t) = \sin t \cdot x(t)$

$\vec{x}(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}$.

$\vec{x}'(t) = \begin{pmatrix} x_1(t) + x_2(t) \\ 2x_1(t) - 2x_2(t) \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 2 & -2 \end{pmatrix} \cdot \vec{x}(t) + \begin{pmatrix} \sin t \\ e^t \end{pmatrix}$